I

EARTH, LINES & DEFINITIONS

The Earth

Navigation is a field of study that focuses on the process of monitoring and controlling the movement of an aircraft from one place to another.

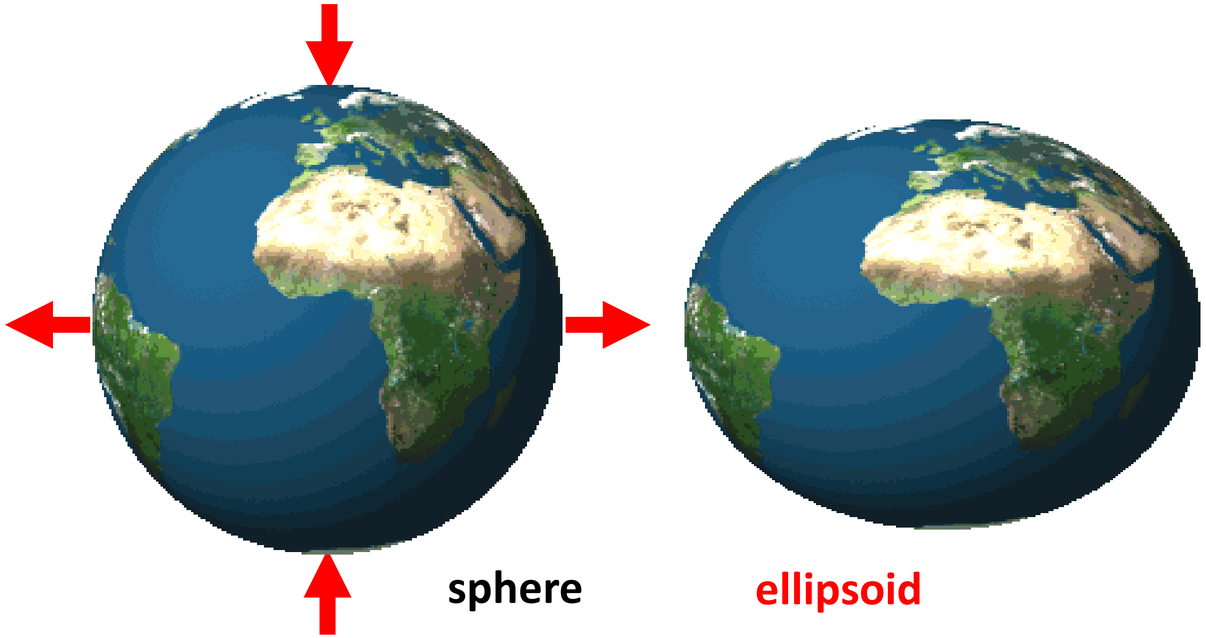
Since we navigate on Earth, it is important to study the Earth shape and to define the location references.

To prepare a navigation, we need a reduced representation of the planet Earth.

The Earth is a sphere on space with 40.000 km circumference and 6.371 km radius, but not a perfect sphere for two reasons:

- The fact the Earth rotates around itself, make the centrifugal force to expand its radius at its equator and to flatten its shape at the pole. This flattening is called the **Earth Compressibility and its value is 0,3% or 1/298**. More simply put, the Earth's polar diameter is 23 nautical miles or 43 km less than its equatorial diameter.
  This shape is known as an **ellipsoid**.

- The terrain or the relief on Earth is not the same (Mountains, oceans, valleys, etc.) which make the imperfections of Earth sphericity.

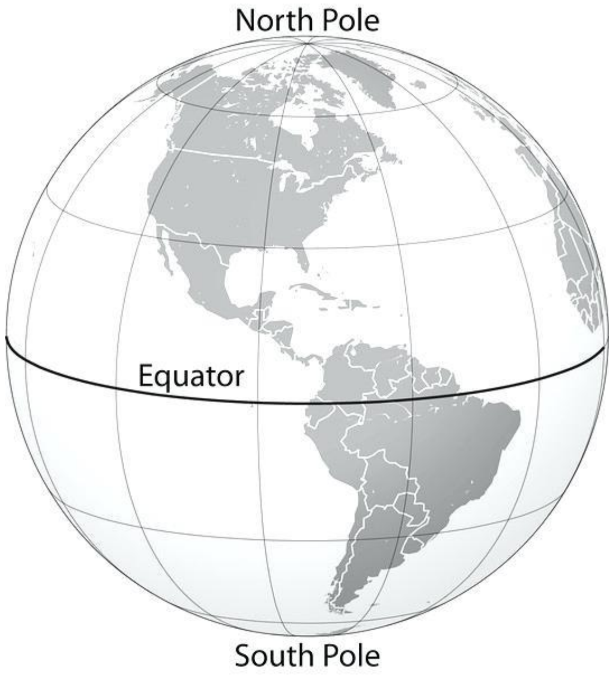


Many systems proposed different model of the Earth representations. The one kept and used by ICAO for navigational purpose, it's the **World Geodetic System of 1984**, known as **WGS 84**, composed with a coordinate system and an ellipsoid of reference. "Geodetic" comes from geodesy, the science of measuring the size and shape of Earth.

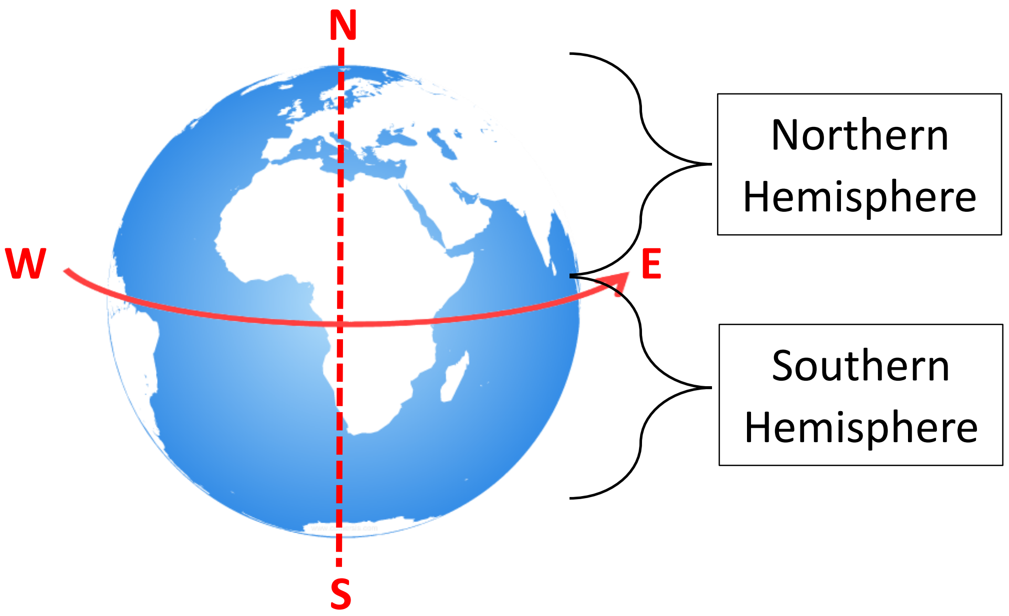
Latitudes and Longitudes

To navigate and to spot positions, it is important to allocate coordinates to each location for referencing.

From space, the Earth doesn't have an up and down side, nor a left or right side. It is by pure convention that the Earth is represented and drawn like the following



The Earth rotates around its polar axis that crosses the North Pole and the South Pole and perpendicular to the equator, and from West to East. The Equator cutes the Earth in two hemisphere, the Northern Hemisphere and the Southern Hemisphere.



We have here to start the referencing, **cardinal points: North, South, East and West.**

A direction toward the North Pole is North, toward the South Pole is South, to the same direction of the Earth's rotation is East and opposite direction to the Earth's rotation is West.

Note: The Earth seems spinning counter clockwise when seen from the North Pole (and clockwise when seen from the South Pole)

Latitudes

The first reference consist to say where the location (X) is compared to the equator.

A straight line is plotted through the location X to the centre of Earth, then the angle between the equator and that line is measured. Example here, the angle is 50° and toward the North Pole. This angle is called Latitude and its value is **50°N**

If the same angle had been measured toward the **South Pole**, so its value would be **50°S**.

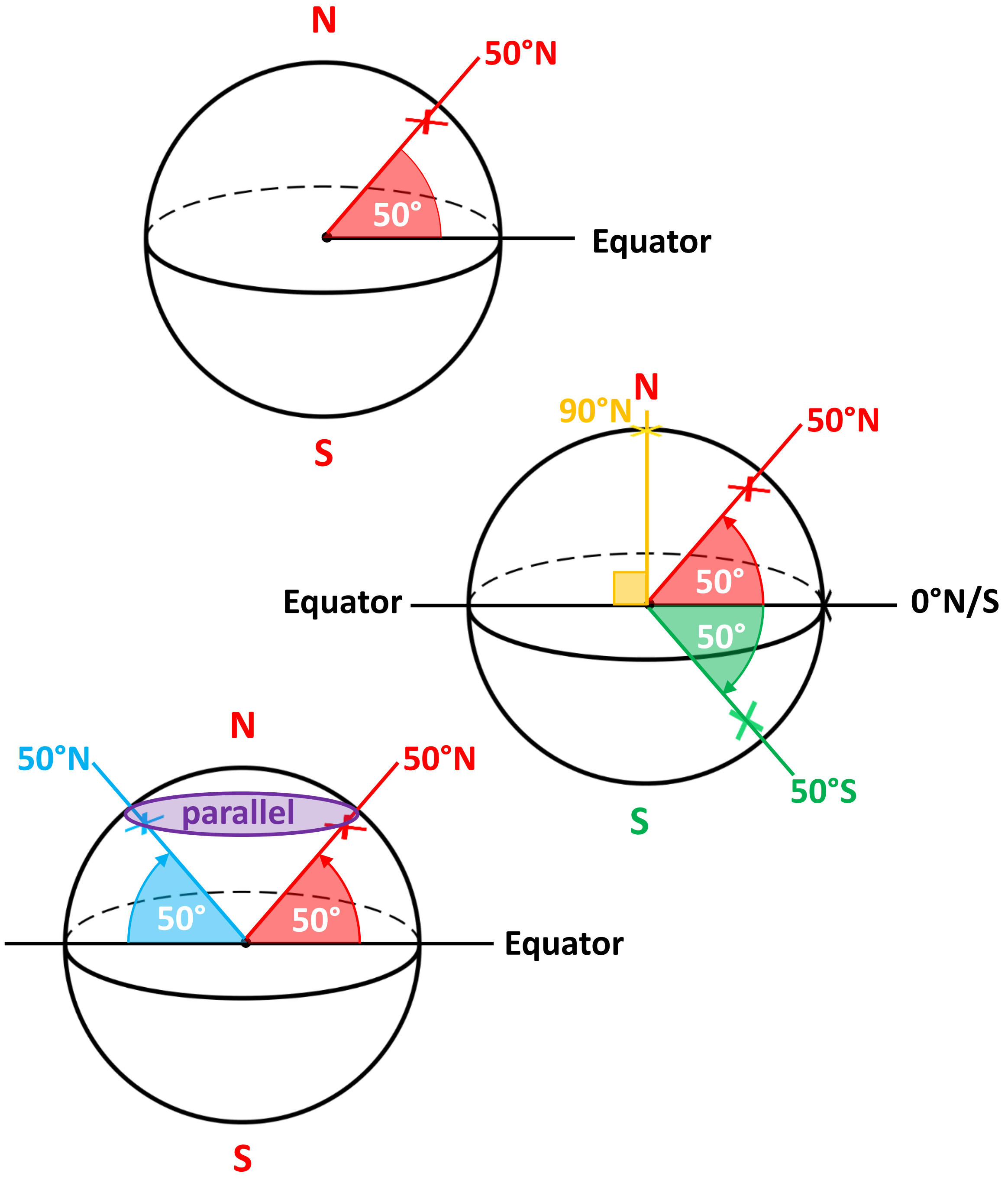
If the location **X** had been at the equator, so its value would be **00°N/S**

If the location X had been exactly at the North Pole, so its value would be 90°N and for the South Pole it would be 90°S.

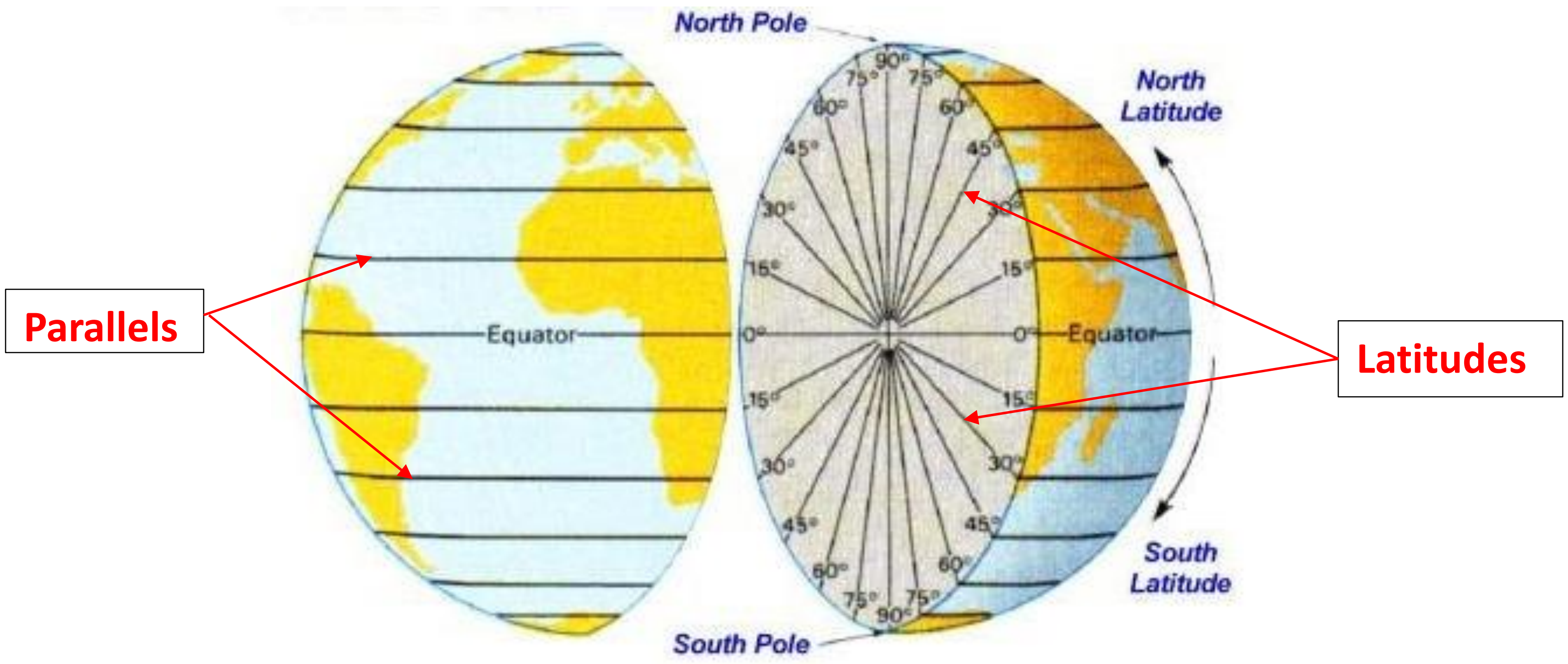
The latitude is comprised between 00°N/S to 90°N/S (so always written with 3 digits)

Notice that, the same angles could be measured on any circle perpendicular to the polar axis.

We call this circle connecting the same latitudes, a **parallel of latitude** or simple, a **parallel**.



So, a **Latitude**, it's the angle measured between the equator and the line crossing that location to centre of Earth, and its value is comprised between 90°N and 90°S. Although a **parallel** is the circle connecting the same latitudes.



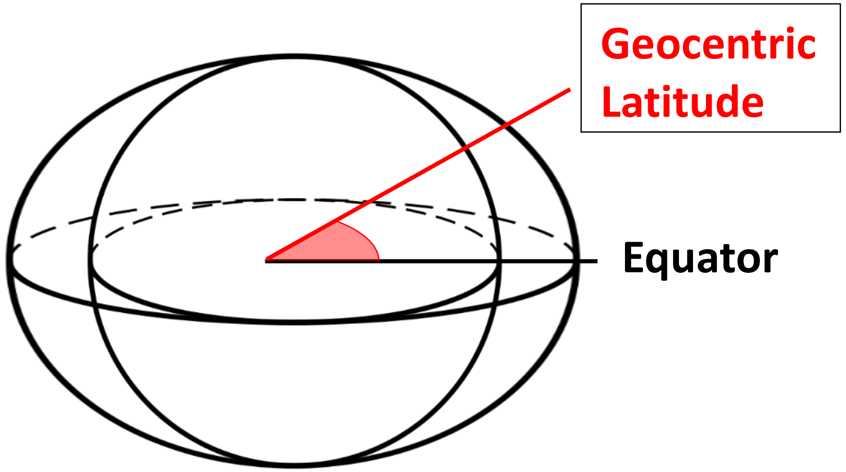
Note that, when you **keep** walking or tracking a **parallel**, the **direction is** either **East or West**.

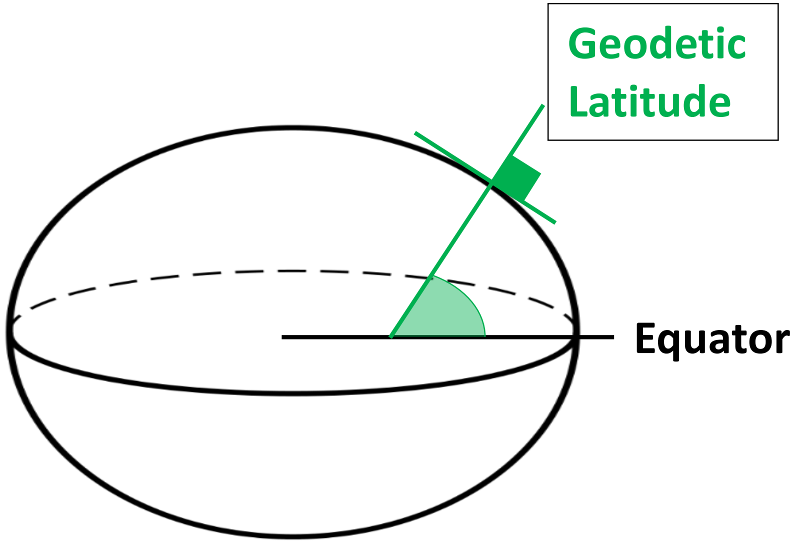
Geocentric and Geodetic latitude

Remember, it has been stated that the Earth is an ellipsoid and not a sphere (on the drawing, the ellipsoid is exaggerated to have a better view).

When we defined the latitude, the angle was measured between the equator and the line crossing a location to the centre of Earth, is latitude measured at the centre of Earth is called a **Geocentric Latitude**.

However the WGS84 assumes that the Earth is perfect sphere, and measures the angle between the equator and the perpendicular to the Earth surface on that location to the equator. This angle is the **Geodetic Latitude**.



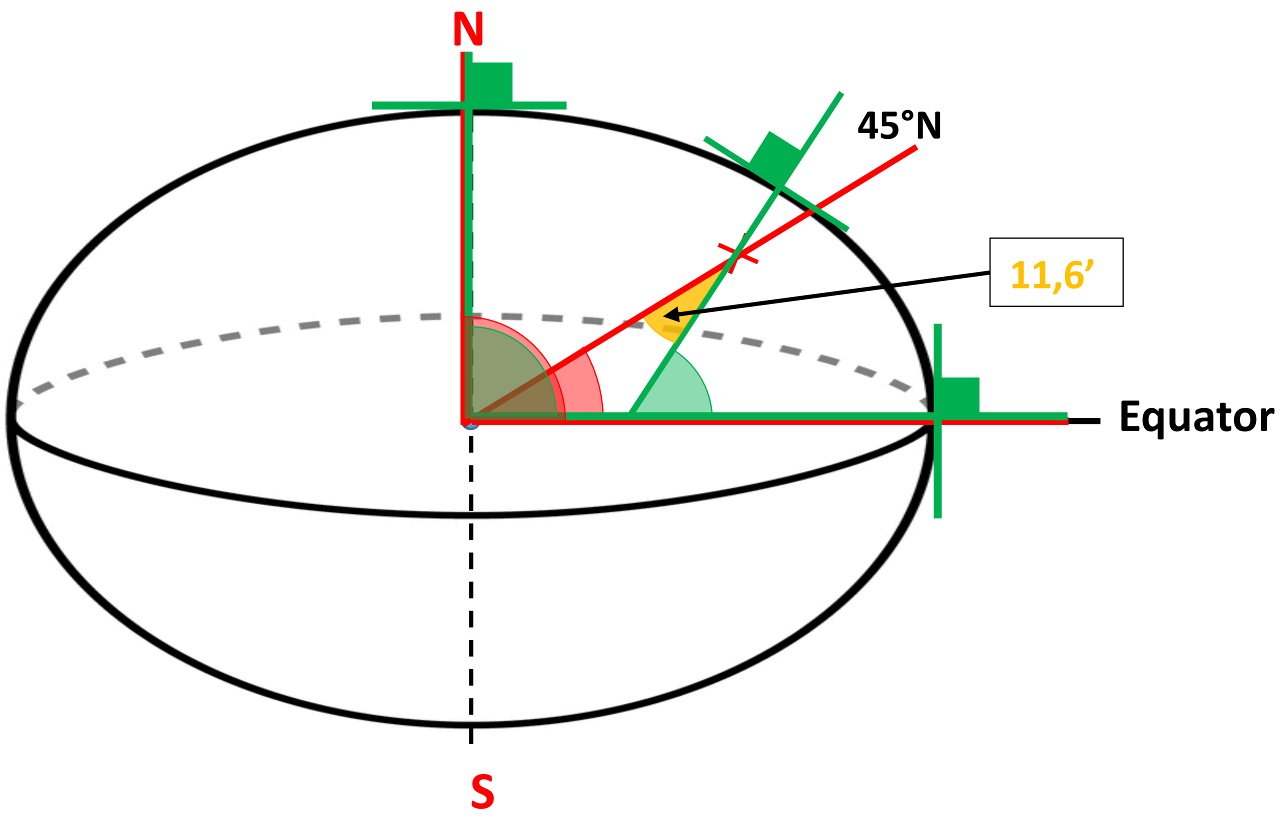


Geocentric and Geodetic latitude

The geodetic latitude is used to keep the measurement of latitude simple. However does that mean that the geodetic latitudes, in real, are wrong? Not really, because:

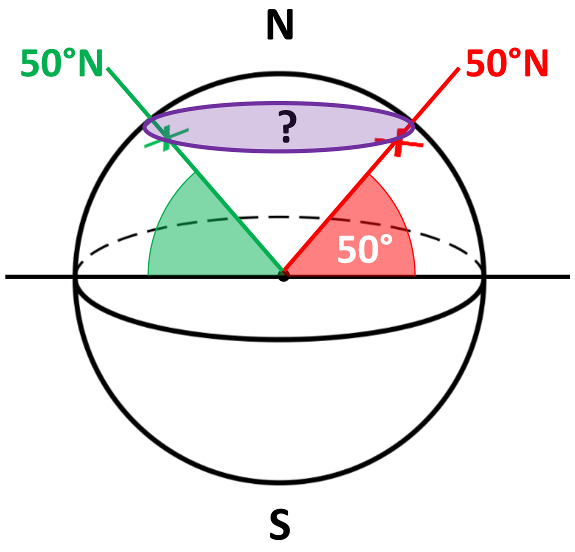
- If we compare the geocentric latitude at the Poles and at the Equator, they are the same.
- The difference starts to be seen when moving away from the Equator and the Poles and reach its maximum at the latitude 45°N and 45°S (equidistant point between the Equator and the Poles).

  Where the difference in angle is $\frac{11{,}6°}{60}$ or 11.6' of arc, which is very insignificant and not considered as an error.

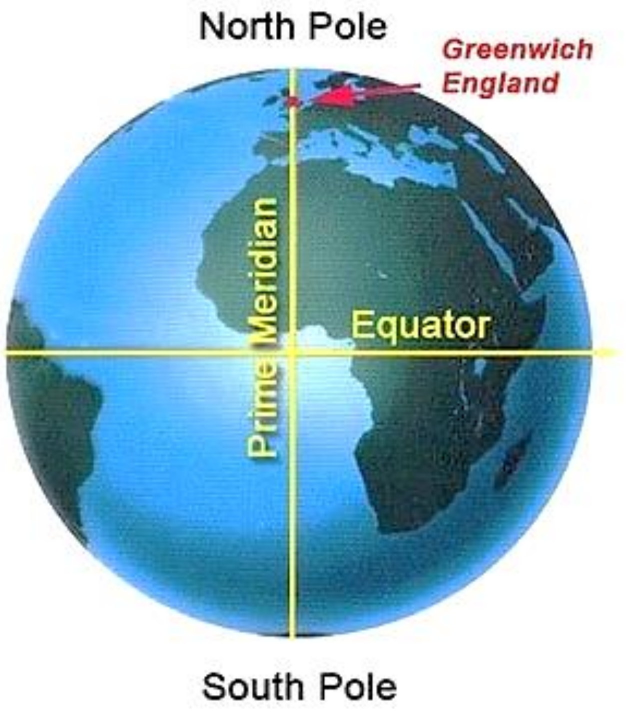


Longitudes

A latitude isn't enough to define a precise location on Earth, remember it exist a parallel of latitude, a circle, and now we need to define the position on that circle.

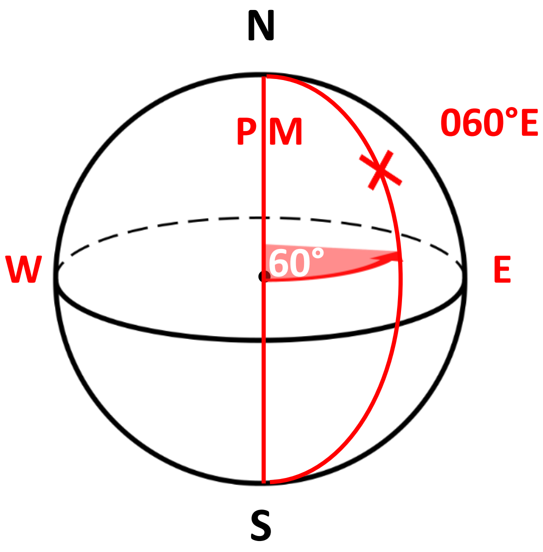


The same method will be used, however now the circle of reference isn't the equator, but the circle that crosses the Poles via the Greenwich observatory located in the United Kingdom. The half circle between the Poles via the Greenwich Observatory is called the **Prime Meridian (PM)**.



The second reference now consist to say where the location (X) is compared to the Prime Meridian.

A half circle is plotted between the poles through the location (**X**), then the angle between the Prime Meridian and that half circle is measured. Example here, the angle is 60° and toward the East. This angle is called Longitude and its value is **60°E**

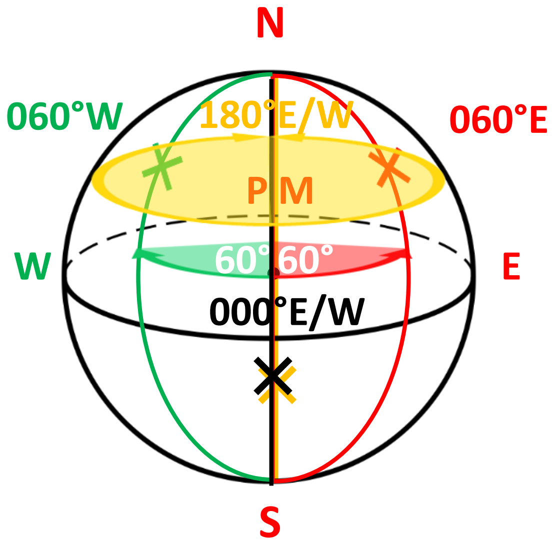


Longitudes

If the same angle had been measured toward the West, so its value would be 060°W.

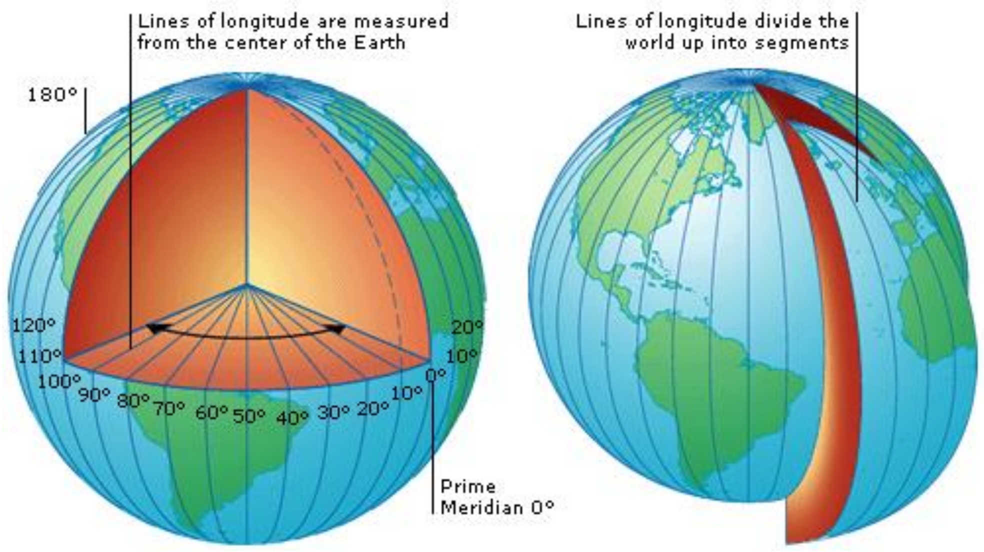
If the location **X** had been at the Prime Meridian, so its value would be **000°E/W**

If the location X had been exactly at the half circle opposite to the Prime Meridian (Greenwich anti-meridian), so its value would be 180°E/W.



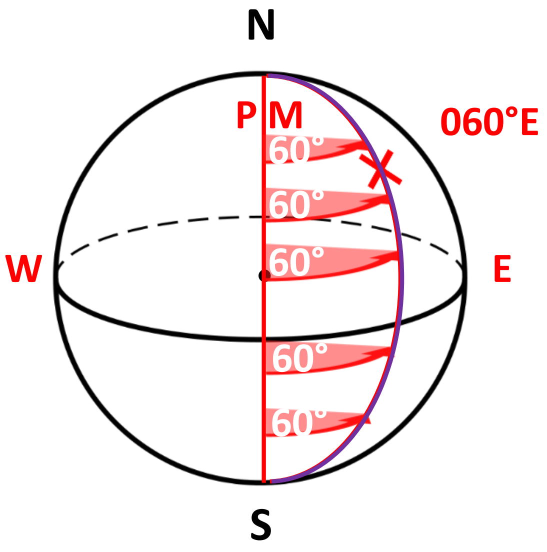
The longitude is comprised between 000°E/W to 180°E/W (so always written with 3 digits)

For a better view, this how the longitudes are measured:

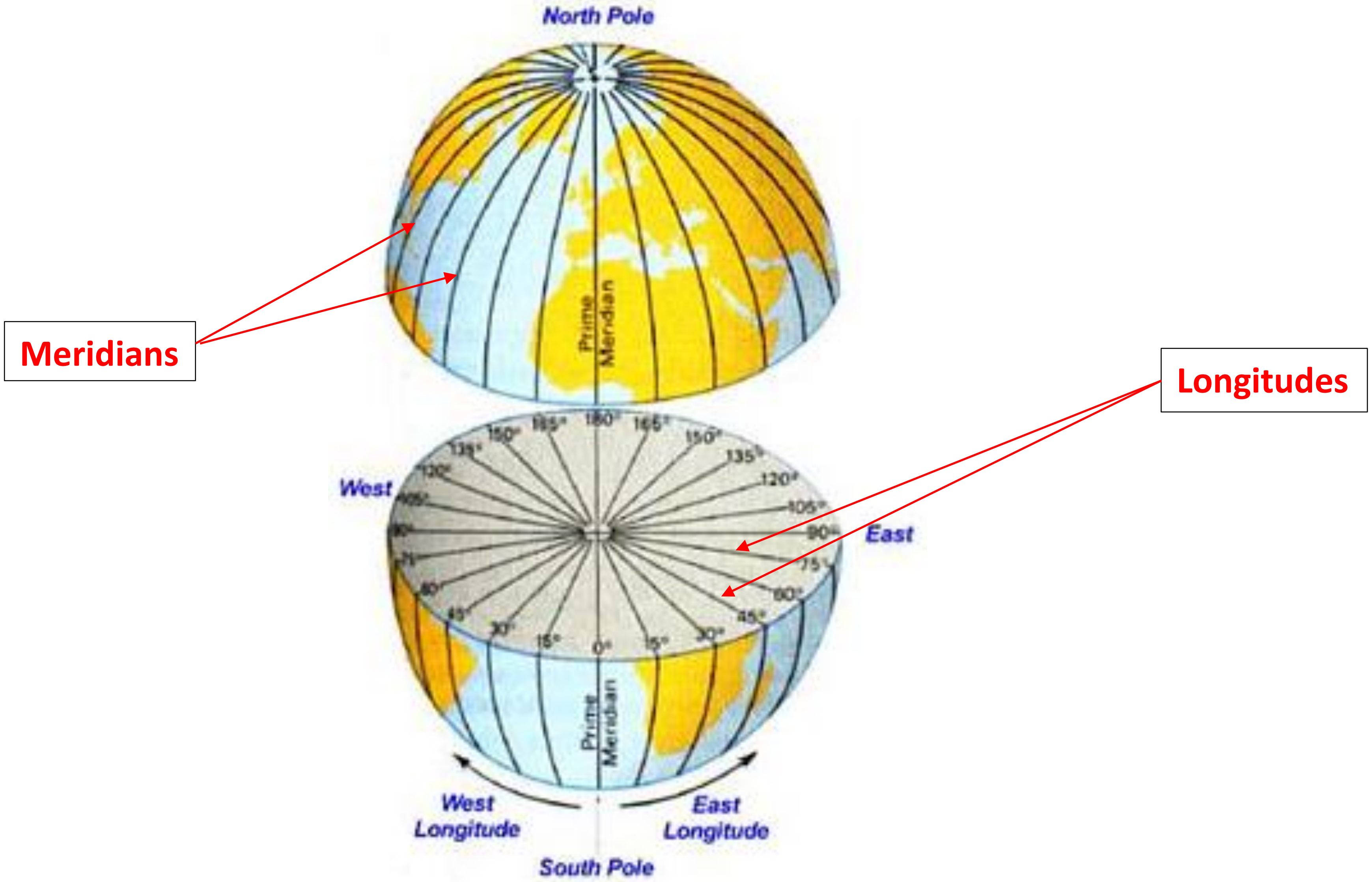


Notice that, the same angles could be measured at any latitude.

We call this line connecting the same longitudes, a **meridian of longitude** or simple, a **meridian**.



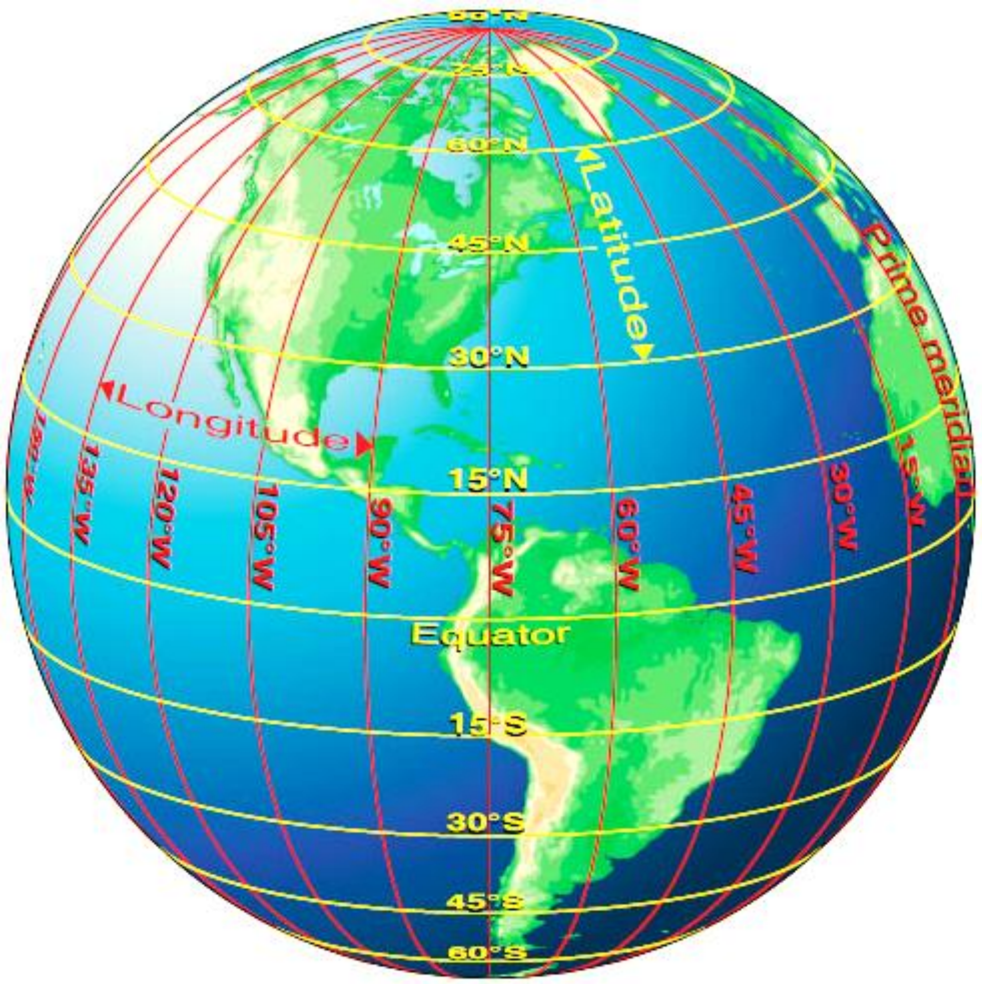
So, a **Longitude**, it's the angle measured between the Prime Meridian and the half circle between the Poles via that location, and its value is comprised between 0°E/W and 180°E/W. Although a **Meridian** is the line connecting the same longitudes.



Note that, when you **keep** walking or tracking a **Meridian**, the **direction is** either **North** or **South**.

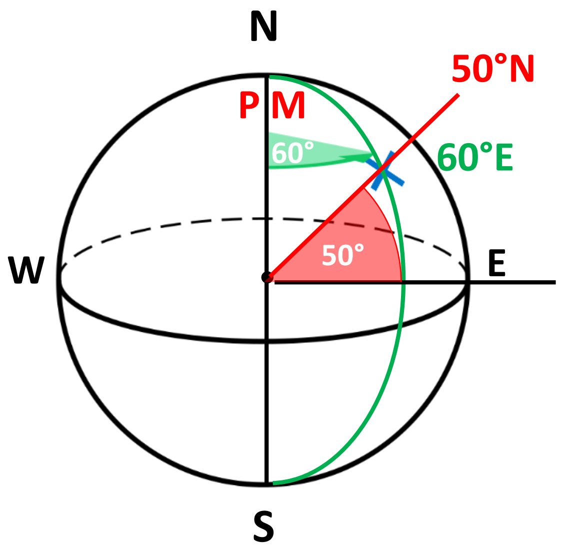
Graticule

The network formed on a map or the surface of a globe by the Prime Meridian, the meridians, the Equator and the parallels of latitudes is called the Graticule.



Geographical Coordinates

Now that we've allocated a latitude and a longitude to that point (X), we have now the geographical coordinates for that point and now we are able to find where it is on Earth. **The Geographical Coordinates of this point are 50°N; 060°E**. Other format exist such as 50N060E, or N50; E060, etc

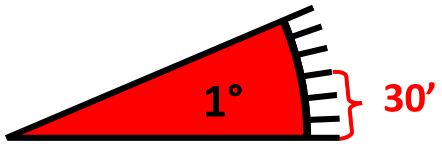


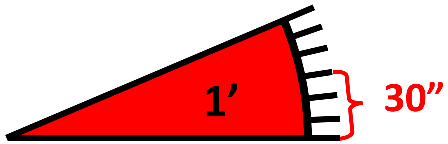
To be more precise, we can split the angles in fractions of degrees.

1° of arc (degree of arc) can be split into 60' of arc (minutes of arc)

And 1' of arc can be split in 60" of arc (seconds of arc)

1° = 60' **1' = 60"** **1° = 3600"**





Conversion

$$X^\circ = (X \times 60)'$$

$$X' = (X \times 60)''$$

$$X^\circ = (X \times 3600)''$$

$$X' = \left(\frac{X}{60}\right)^\circ$$

$$X'' = \left(\frac{X}{60}\right)'$$

$$X'' = \left(\frac{X}{3600}\right)^\circ$$

h/min °/'	min/sec '/"
x60	**=**
0.1	6
0.2	12
0.3	18
0.4	24
0.5	30
0.6	36
0.7	42
0.8	48
0.9	54
1.0	60

Exercises:

What is the Change of Latitude (chLAT or D.LAT) or Change of Longitude (chLONG or D.LONG) between the following positions and the shortest direction (North, South, East or West):

a. 52°N 35°W to 39°N 35°W

b. 45°N 075°W to 45°N 125°W

c. 74°20’S 45°32’W to 34°30’S 45°32’W

d. 40°20’S 001°20’E to 40°20’S 004°20’W

e. 71°20’S 120°30’E to 86°45’N 120°30’E

f. 00,0°N/S 162,6°W to 00,0°N/S 140,7°E

g. 49°N 178°E to 60°S 178°E

h. 32,1°S 179,5°E to 32,1°S 150,6°E

i. 30°42’N 171°30’E to 30°42’N 175°18’W

j. 80°N 120°E to 80°N 60°W

The correction is in the next page…

Exercises *(correction)*:

What is the Change of Latitude (chLAT or D.LAT) or Change of Longitude (chLONG or D.LONG) between the following positions and the shortest direction (North, South, East or West):

a. 52°N 35°W to 39°N 35°W

ChLAT 13° toward South

b. 45°N 075°W to 45°N 125°W

ChLONG 50° toward West

c. 74°20'S 45°32'W to 34°30'S 45°32'W

ChLAT 39°50' toward North

d. 40°20'S 001°20'E to 40°20'S 004°20'W

ChLONG 5°40' toward West

e. 71°20'S 120°30'E to 86°45'N 120°30'E

ChLAT 158°05' toward North

f. 00,0°N/S 162,6°W to 00,0°N/S 140,7°E

ChLONG 56.7° toward West

g. 49°N 178°E to 60°S 178°E

ChLAT 109° toward South

h. 32,1°S 179,5°E to 32,1°S 150,6°E

ChLONG 28.9° toward West

i. 30°42'N 171°30'E to 30°42'N 175°18'W

ChLONG 13°12' toward East

j. 80°N 120°E to 80°N 60°W

(∑co-latitudes = 10°+10°) ChLAT 20° via the North Pole

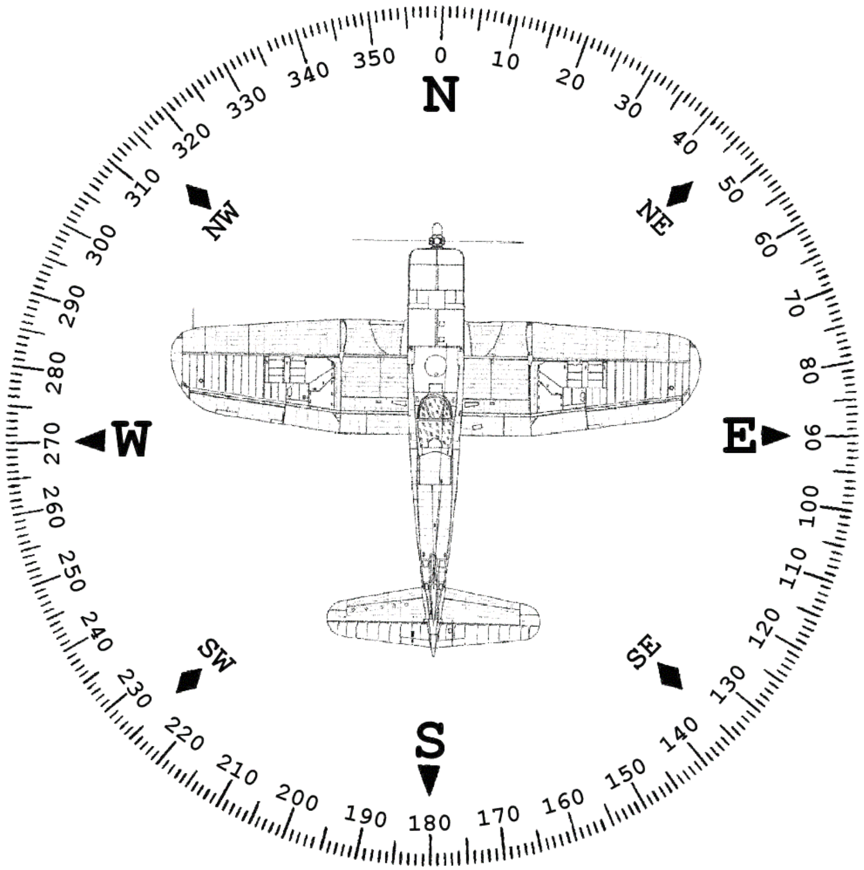
***Tips*:**

- If the two locations are on the same longitude and same hemisphere (North or South), simply subtract the lowest latitude value from the highest latitude value to find the ChLAT (eg. 52°N 35°W to 39°N 35°W, ChLAT = 52° – 39° = 13°)
- If the two locations are on the same latitude and same hemisphere (East or West), simply subtract the lowest longitude value from the highest longitude value to find the ChLONG (eg. 45°N 075°W to 45°N 125°W, ChLONG = 125° – 75° = 50°)
- If the two locations are on different latitudes (North and South) but same longitude, simply sum up the two latitude values to find the ChLAT (eg. 71°S 120°E to 86°N 120°E, ChLAT = 71° + 86° = 157°)
- If the two locations are on different longitudes (East and West) but same latitude, simply sum up the two longitude values to find the ChLONG (eg. 40°S 001°E to 40°S 004°W, ChLONG = 1° + 4° = 5°)
- If the two locations are on different longitudes (East and West) but same latitude, and summing up the two longitude values results in a ChLONG >180°, so subtract this result from 360° to obtain the shortest ChLONG. (eg. 20°N 162°W to 20°N 140°E, ChLONG = 162° + 140° = 302° >180°. Shortest ChLONG = 360° – 302°= 58°)
- When two locations are on different longitudes (East and West) and same latitude, if the sum of their longitude values is less than 180°, so the shortest direction is the designation of the second point, and if the sum of their longitude values is more than 180°, so the shortest direction is the designation of the initial point (eg1. 40°S 001°E to 40°S 004°W, ∑LONGs = 5° <180°, so the shortest direction is West. Eg2. 40°S 170°E to 40°S 160°W, ∑LONGs = 330° <180°, so the shortest direction is East)
- When two locations are separated by 180° ChLONG, the shortest distance is via the pole and to obtain the ChLAT, simply sum up their co-latitudes (90°-Latitude). (Eg. 75°N 120°E to 75°N 60°W, ChLAT = [90-75]° + [90-75]° = 15° + 15° = 30°)
- To find the anti-meridian to a meridian, their value must be equal to 180° and the designation East becomes West and vice versa. (Eg. the anti-meridian to the meridian 40°E is 140°W because 40°+140°+180°)

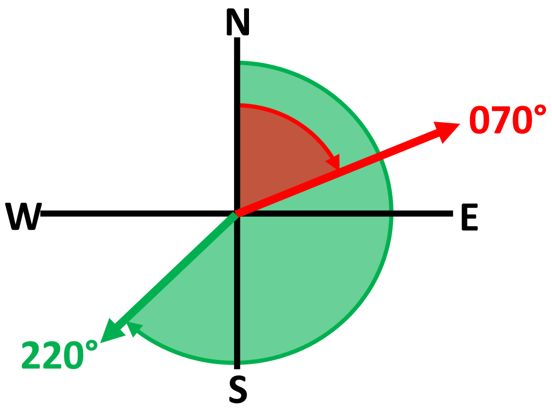
Direction

Now that we understood that the meridians connect the north to the south, we can speak about the direction.

A direction is always compared from the direction to the north, in other words, compared from the actual meridian on which you are. A direction is an angle measured clockwise from the North, and its value is comprised between 000° and 360°.



Travelling to the North, the direction is 360°, the East is 090°, the South is 180°, the West is 270°, North-East is 045°, South-East is 135°, South-West is 225°, and North-West is 315°



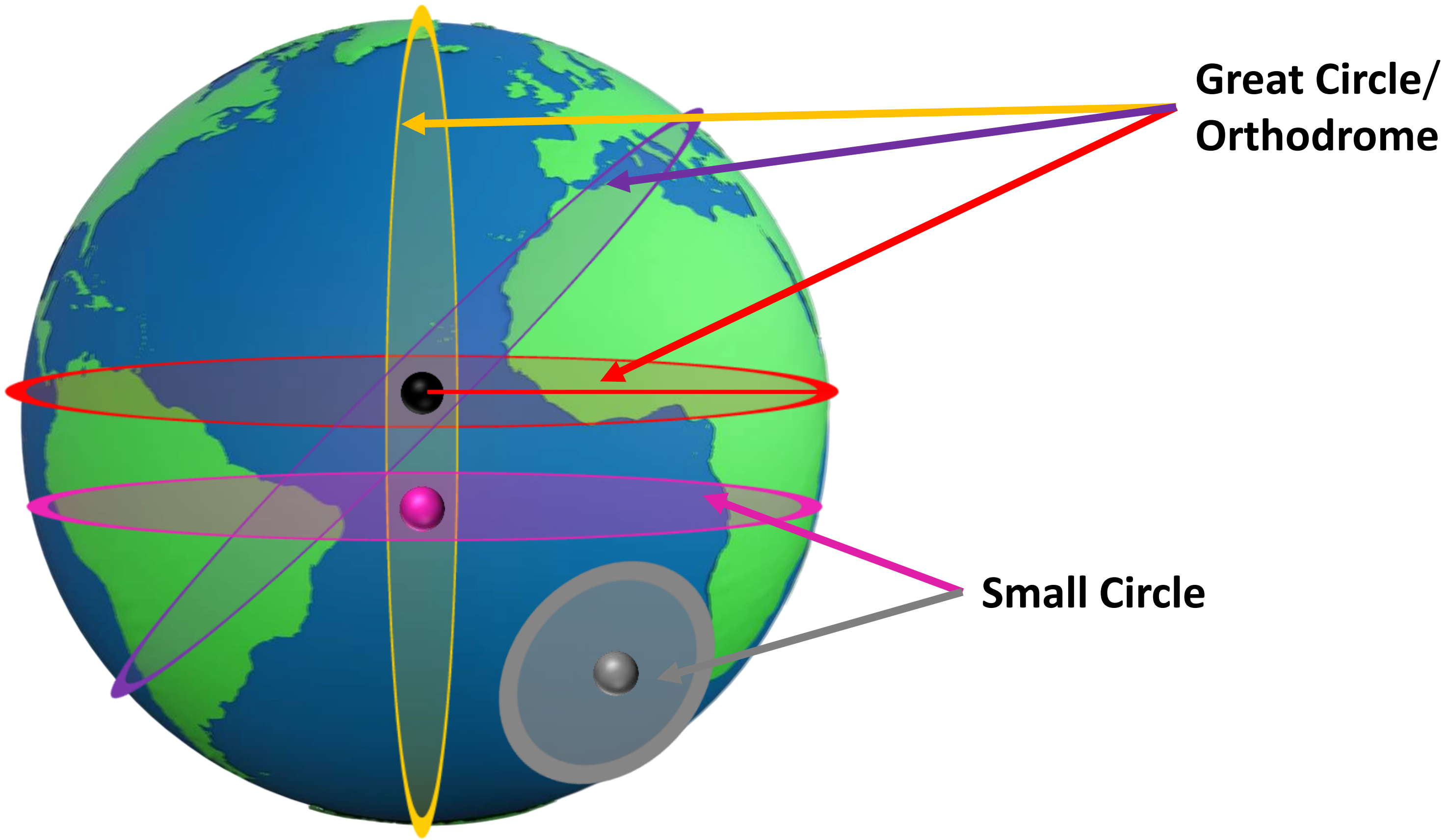
Remaining on the meridian, the direction is either North (360°) or South (180°), and on the parallel the direction is either East (090°) or West (270°).

3) Definitions on the globe

Great Circle/Orthodrome

A circle on the surface of the earth whose centre and radius are those of the earth itself is called a **Great Circle** or **Orhtodrome**. It is called 'great' because a disc cut through the earth in the plane of the Great Circle would have the largest area that can be achieved.

The **Equator**, and all **meridians with their anti-meridians** are **Great Circle**. These are known as "natural" Great Circle, because they are obvious and easy to identify. However there is an infinity of **Great Circles**.



Small Circle

A circle on the surface of the earth whose centre and radius are not those of the earth is called a Small Circle.

All **parallels**, except the **Equator**, are small circles

Great Circles Vertices

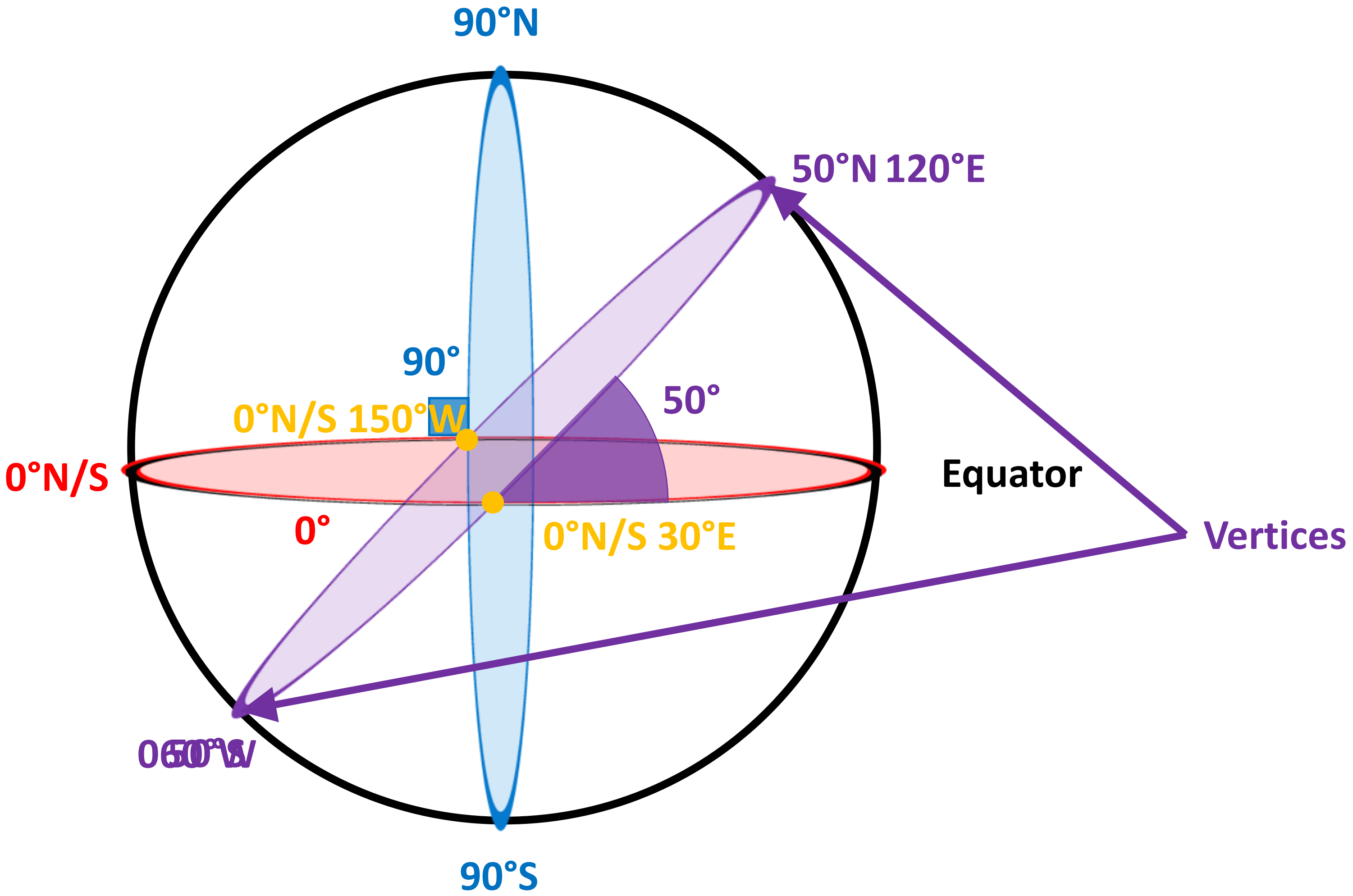
The vertices of a Great Circle are the highest latitudes reached by a Great Circle

The northern vertex of a Great Circle is simply the most northerly point on that Great Circle. Similarly, the southern vertex is the most southerly point on the Great Circle. The vertices are antipodal.

The vertices lie on meridian and anti-meridian and have latitude values of equal value but of opposite sign.

For example here, if the southern vertex of a great circle is **50°S 060°W**, its northern vertex will be **50°N 120°E**.

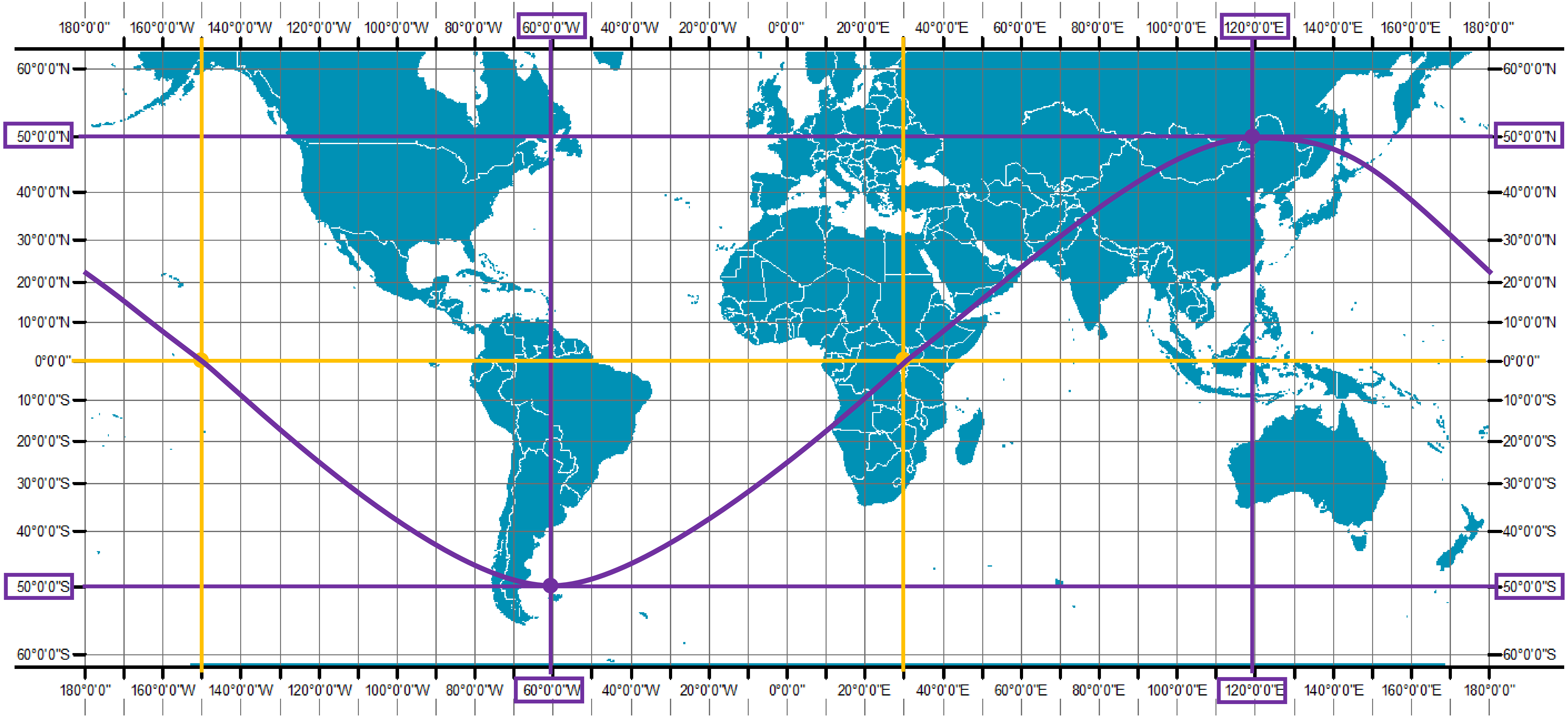
Tip: *To find the anti-meridian to a meridian, their value must be equal to 180° and the designation East becomes West and vice versa. (Eg. the anti-meridian to the meridian 40°E is 140°W because 40°+140°+180°)*



The Great Circle crosses the Equator twice on two points. These point lie on meridian and anti-meridian which are 90° away from the vertices. For example here, the Great Circles crosses the Equator at 0°N/S 030°E and 0°N/S 150°W.

50°S 060°W ←90°→ 0°N/S 030°E ←90°→ 50°N 120°E ←90°→ 0°N/S 150°W ←90°→ 50°S 060°W…

If we try to reproduce the previous **Great Circle**, by location the vertices and the points where it cuts the Equator, the **Great Circle** will take the shape of a sinusoidal curve



Question ID: 610638

A Great Circle Track crosses the equator at 30°W has an initial track of 035°T. It's highest or lowest North/South point is:

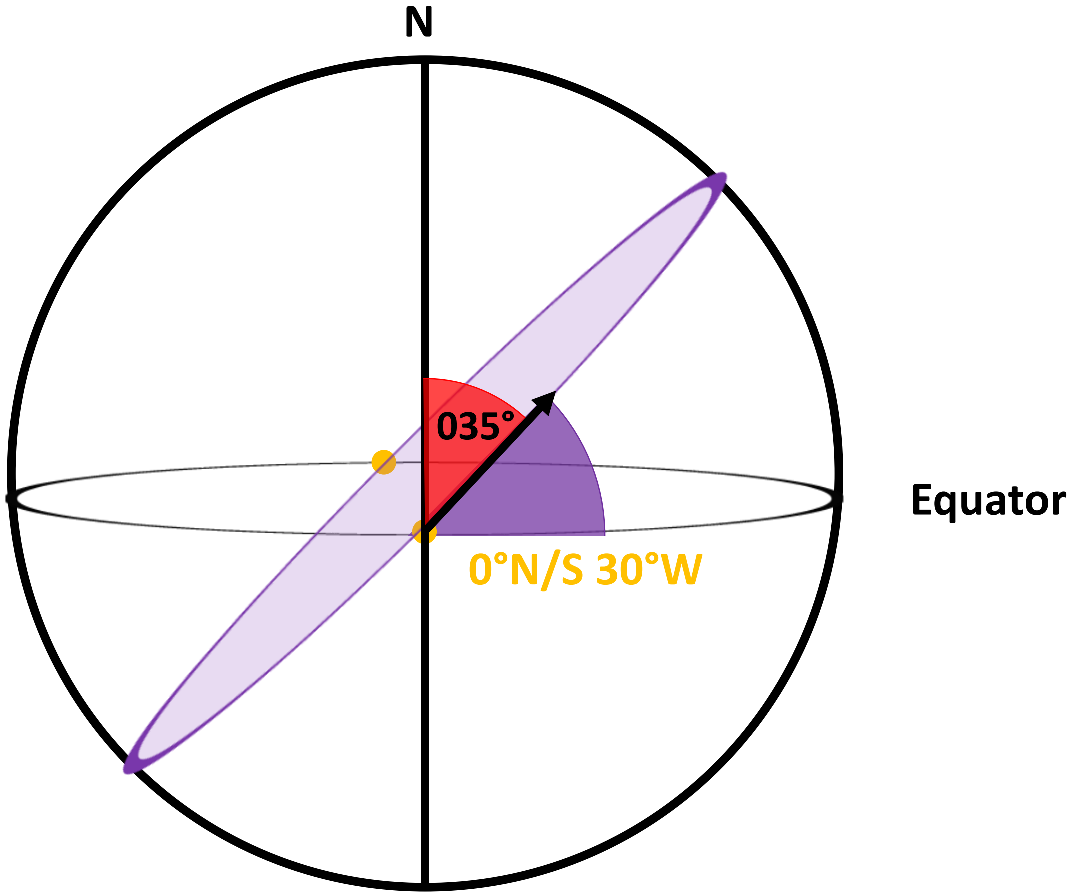
A 35°S 150°E

B 55°N 060°E

C 55°S 060°W

D 35°N 150°W

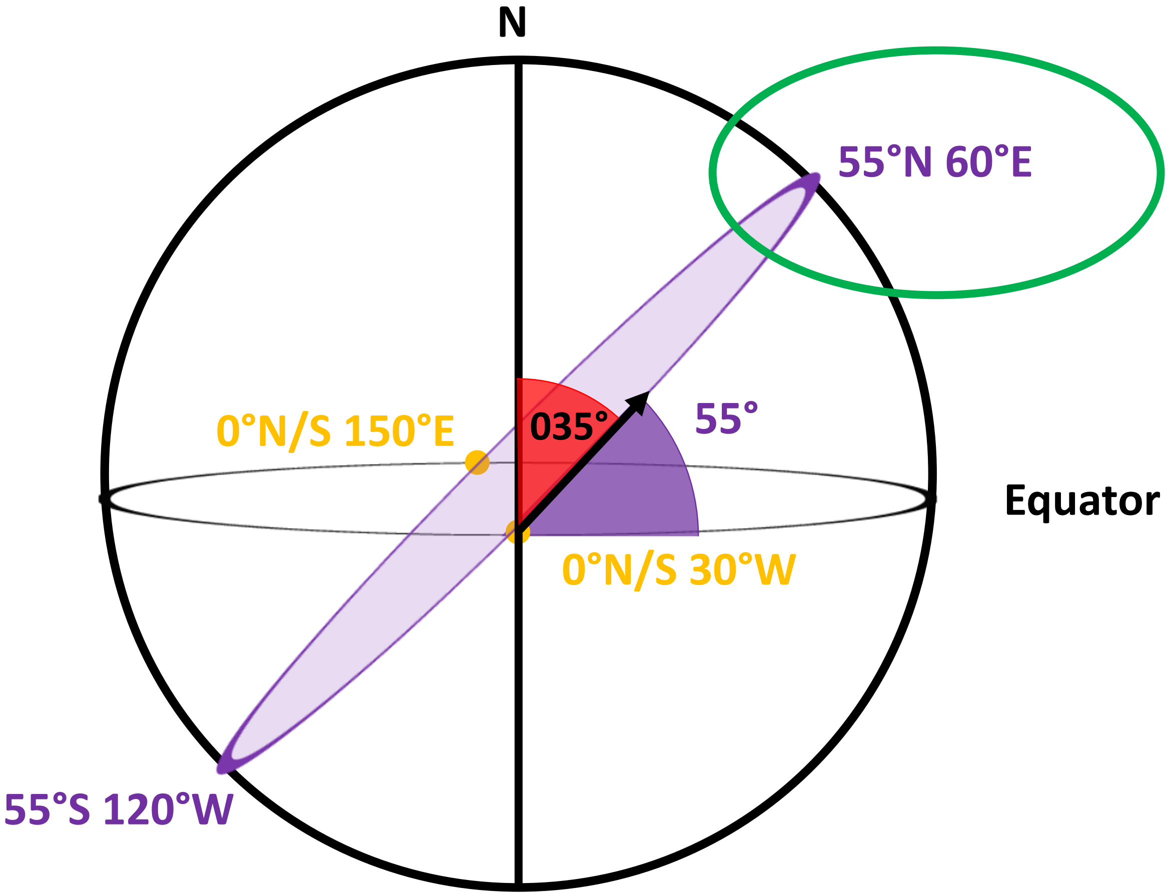
The correction is in the next page…



Question ID: 610638 *(corrected)*

A Great Circle Track crosses the equator at 30°W has an initial track of 035°T. It's highest or lowest North/South point is:

- A 35°S 150°E
- B 55°N 060°E
- C 55°S 060°W
- D 35°N 150°W



We defined the Great Circle, it's because the Great Circle is the shortest distance between 2 points on Earth.

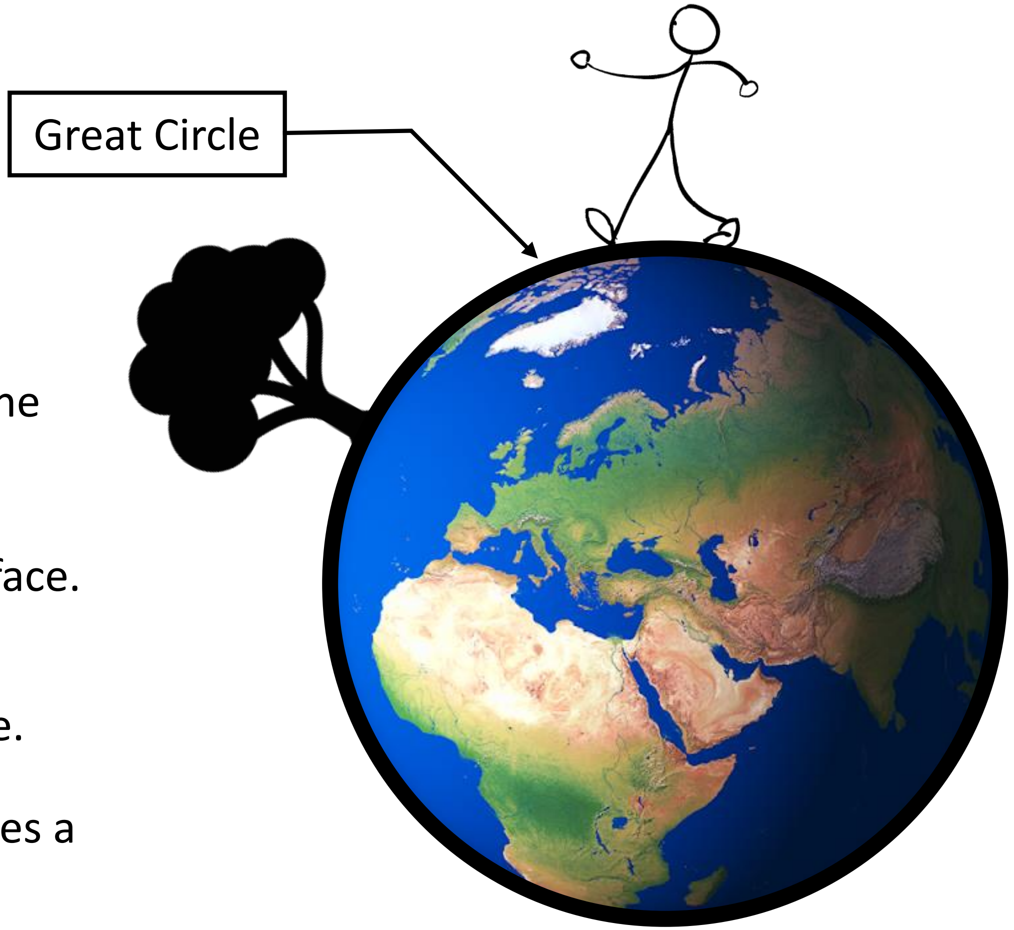
Indeed, the shortest distance between 2 points is a straight line. This is obvious to see when working on 2D plane.

On a 2D plane, between 2 points, it exists one (infinite) straight line

On a 3D sphere, between 2 points, the (infinite) straight line describes a Great Circle around the sphere due to the curved surface.

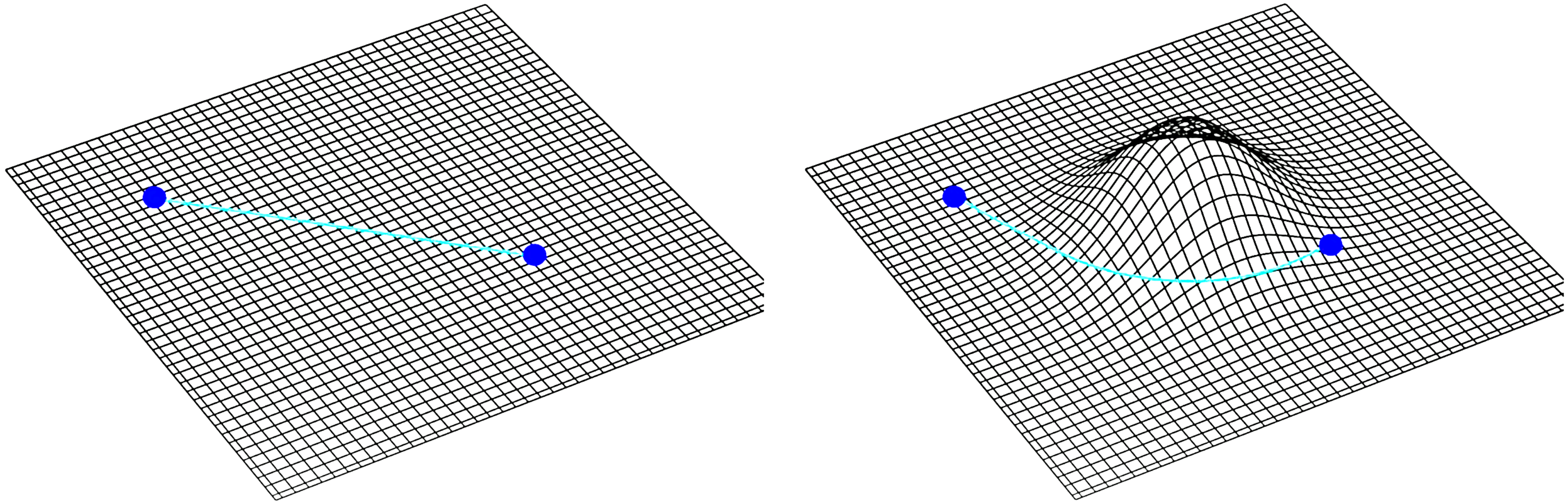
To travel the shortest distance to a point facing you, you walk straight and your trajectory describes an infinite line in a 2D plane.

However, walking straight on a 3D sphere, your trajectory describes a Great Circle around that sphere.



Geodesic

We saw that the straight line, which represents the shortest distance between two points can take different shape than a straight line when the surface isn't flat.



In differential geometry, the curved line representing in some sense the shortest path between two points in a surface is called **geodesic**. It is a generalization of the notion of a "straight line" to a more general setting.

The Great Circle on Earth is also called **geodesic**.

Rhumb Line/Loxodrome

The Rhumb Line or Loxodrome, is a line that crossed all the meridians at the same angle (same direction).

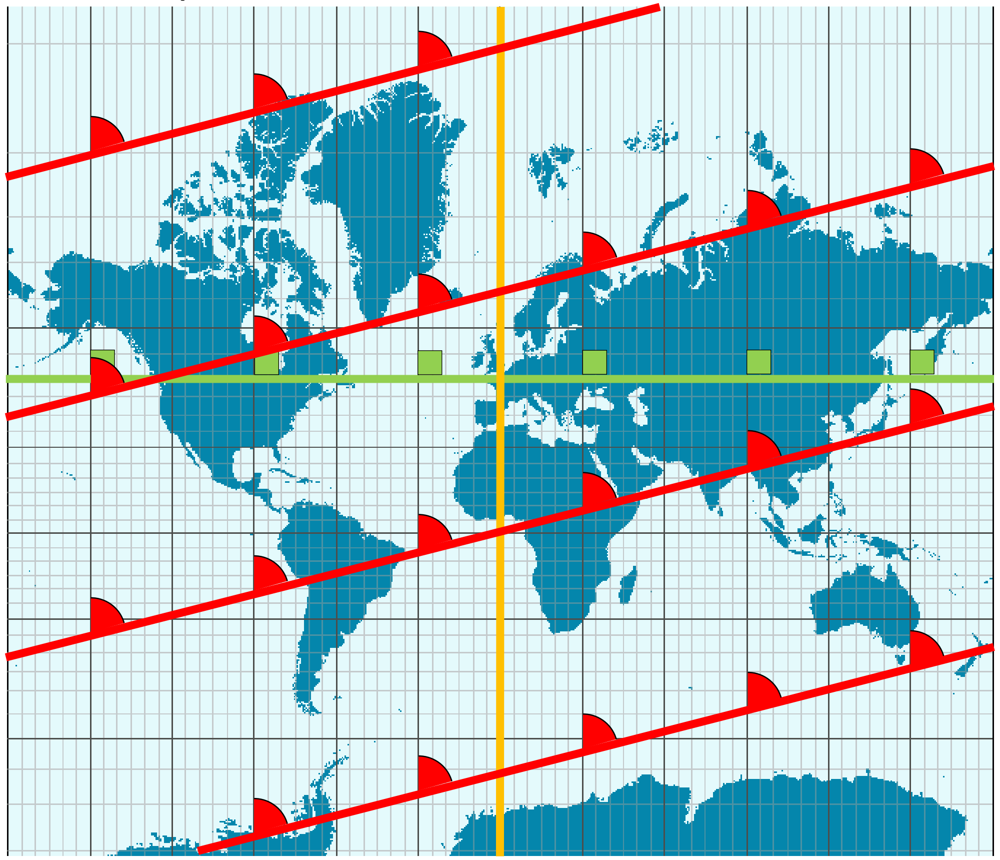
Let's draw this line on a Mercator Map then on the globe. For the moment do not focus on the map projection, it will be discussed later.

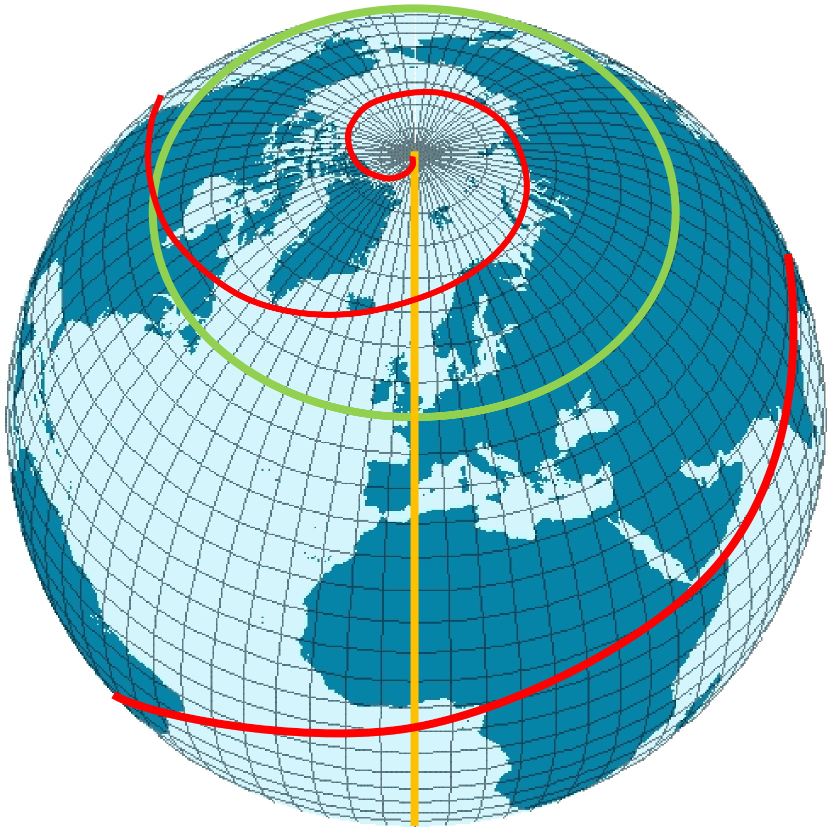
A Meridian **is Rhumb Line**, since it is cutting all other meridians at 0° angle and has one direction: North (360°) or South (180°).

A Parallel **is Rhumb Line**, since it is a circle cutting all meridians at 90° angle and has one direction: East (090°) or West (270°).

Any **line with a constant direction**, **is Rhumb Line**, however if the direction is different from North (360°), East (090°), South (180°) or West (270°), it is describing a **spiral** running between the poles.

Note: The Equator and the Meridians are Great Circles (orthodromes) and Rhumb Lines (loxodromes) at the same time.





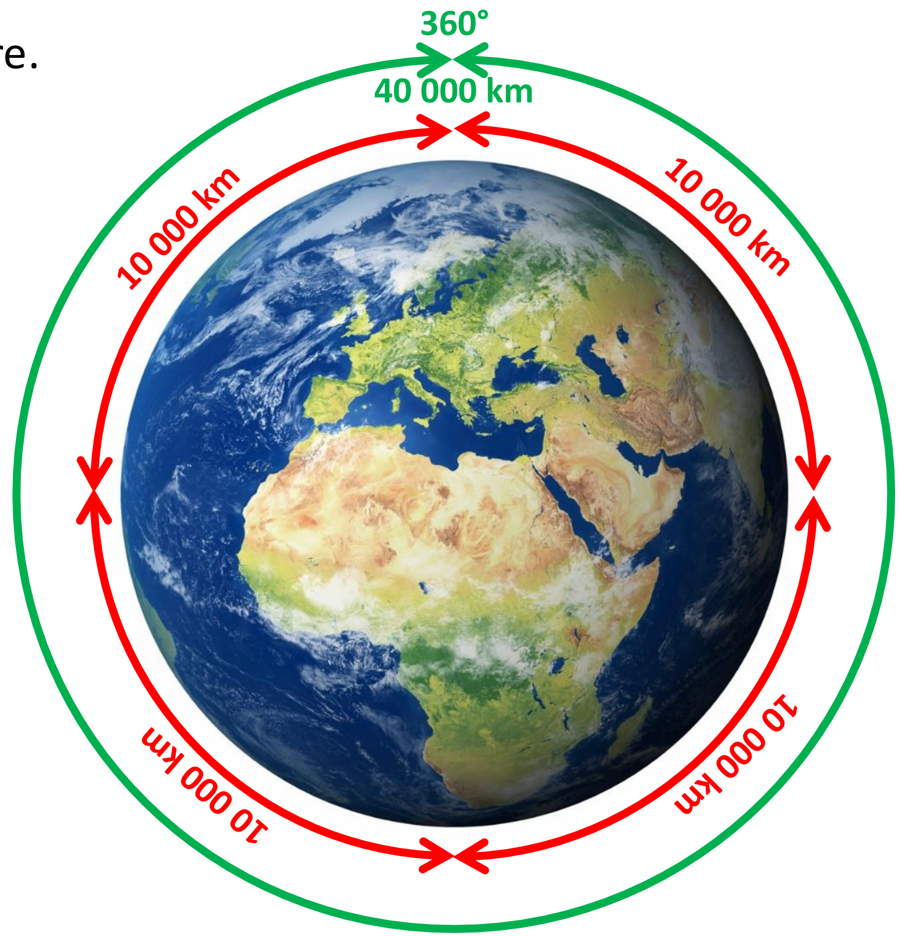
Distance

The Standard International Unit used to measure the distance is metre or kilometre.

The first time that the meter was established it was in 1792 based on ten millionth (1/1.000.000) of half meridian (Pole to Equator).

So it means that the distance between Pole to Equator has arbitrary been set to the value of 1.000.000 meters or 10.000 km

So the average Earth circumference, meaning the distance of a Great Circle, is arbitrary 40.000 km.



If we check how much is a degree of arc on the Great Circle in kilometres

$$\frac{40000\ km}{360°} \approx 111{,}111 km/°$$

Now if we check how much is a minute of arc on the Great Circle in kilometres

$$\frac{111{,}111 km/°}{60'} \approx 1{,}851851 \approx 1{,}852\ km/'$$

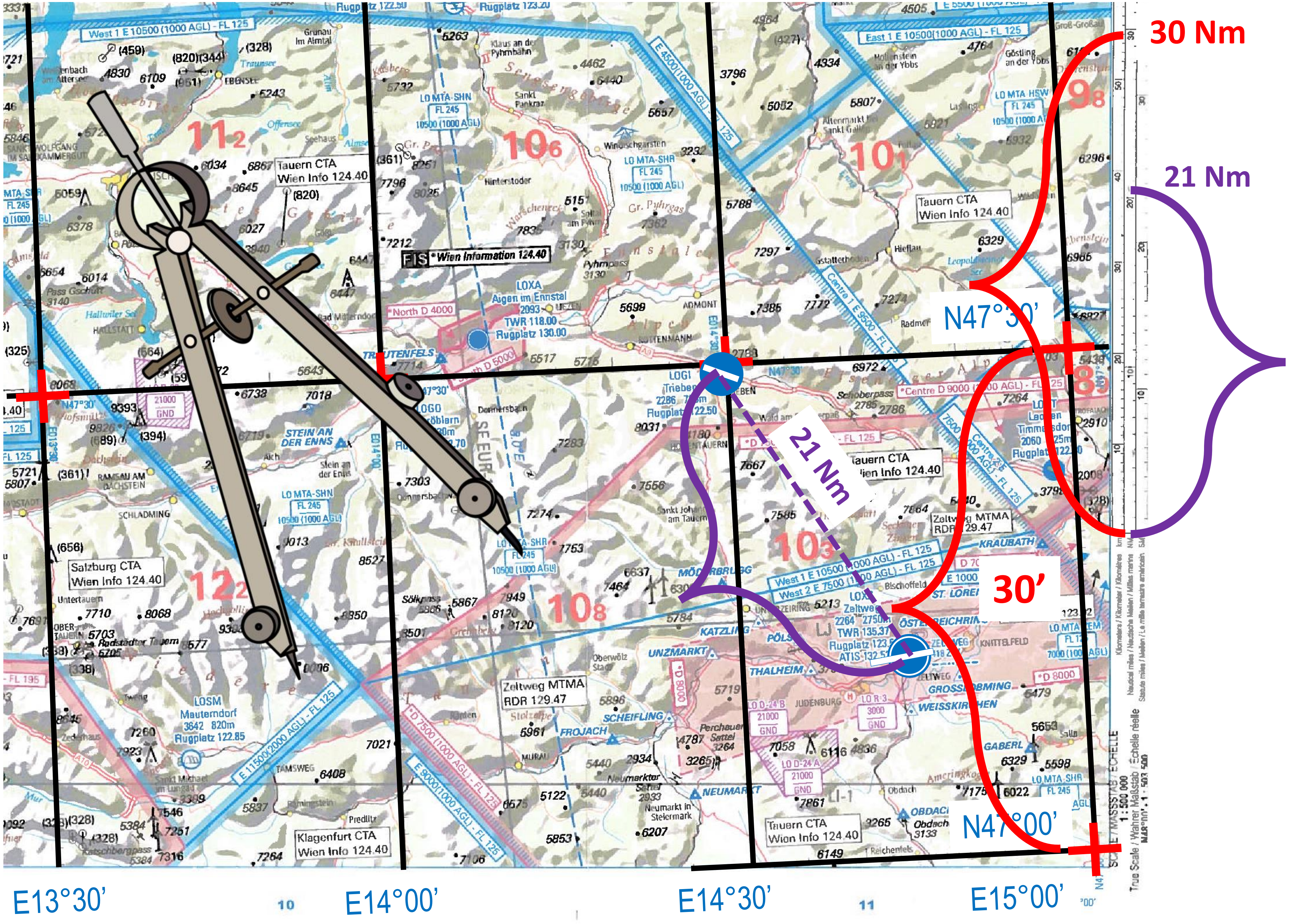
1’ of arc on the Great Circle is 1,852 km, this is the scale reference for navigation and it is called the Nautical Mile (Nm) with:

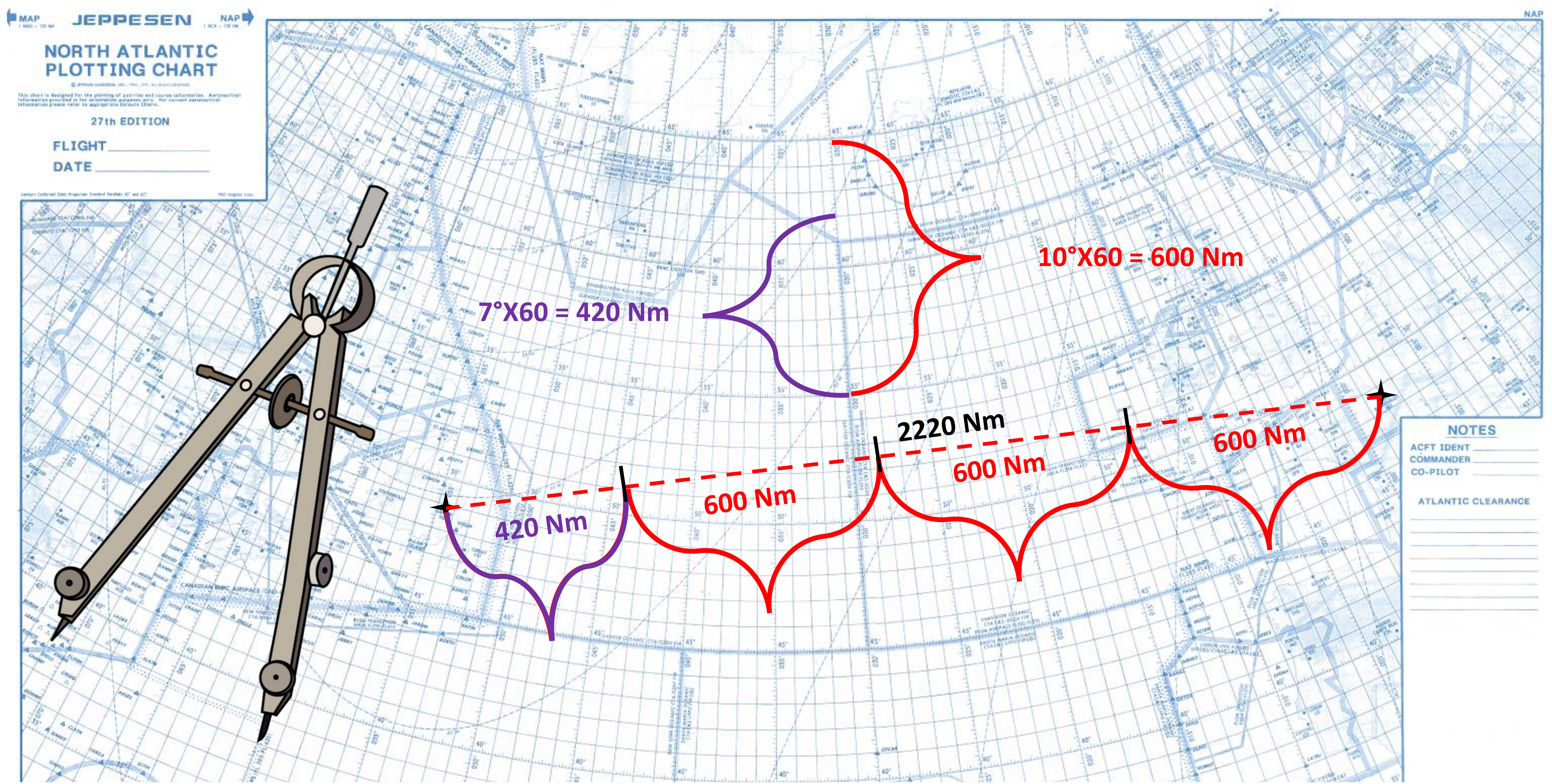
1’ of arc on the Great Circle = 1 Nm **1° of arc on the Great Circle = 60 Nm** **1 Nm = 1,852 km**

The “natural” Great Circles on Earth are the Equator and the Meridians.
This is how it was possible to measure a distance on a map:

The distance between two points is plotted then checked against the Meridians or the Equators to obtain the value.

If we check on any parallel different from the Equator, the value is wrong.





Note: Because of the ellipsoid shape of the Earth, in reality 1' on a meridian is smaller that 1' on the Equator. However, remember that for convenience, WGS84 ignores this fact and assumes that the Earth is perfect sphere.

Exercises 1

What is the distance between the following positions and the shortest direction (North, South, East or West):

For each question, initially verify if the Great Circle criteria are met by checking if it is on the Equator (both latitudes = 0°N/S), or the same meridian (both longitudes are the same) or on the meridians and its anti-meridian (both longitude equal to 180° with one East and one West), then calculate the change in angle in between to convert it to a distance.

a. 52°N 35°W to 39°N 35°W
b. 0°N/S 075°W to 0°N/S 125°W
c. 74°20'S 45°32'W to 34°30'S 45°32'W
d. 0°N/S 001°20'E to 0°N/S 004°20'W
e. 71°20'S 120°30'E to 86°45'N 120°30'E
f. 00,0°N/S 162,6°W to 00,0°N/S 140,7°E
g. 49°N 178°E to 60°S 178°E
h. 0°N/S 179,5°E to 0°N/S 150,6°E
i. 0°N/S 171°30'E to 0°N/S 175°18'W
j. 78°N 120°E to 75°N 60°W

The correction is in the next page…

Exercises 1 *(corrected)*

What is the distance between the following positions and the shortest direction (North, South, East or West):

For each question, initially verify if the Great Circle criteria are met by checking if it is on the Equator (both latitudes = 0°N/S), or the same meridian (both longitudes are the same) or on the meridians and its anti-meridian (both longitude equal to 180° with one East and one West), then calculate the change in angle in between to convert it to a distance.

a. 52°N 35°W to 39°N 35°W

   ChLAT 13° ➢ 13°x60 = 780 Nm

b. 0°N/S 075°W to 0°N/S 125°W

   ChLONG 50° ➢ 50°x60 = 3000 Nm

c. 74°20'S 45°32'W to 34°30'S 45°32'W

   ChLAT 39°50' ➢ 39°x60 +50' = 2390 Nm

d. 0°N/S 001°20'E to 0°N/S 004°20'W

   ChLONG 5°40' ➢ 5°x60 +40' = 340 Nm

e. 71°20'S 120°30'E to 86°45'N 120°30'E

   ChLAT 158°05' ➢ 158°x60 +5' = 9485 Nm

f. 00,0°N/S 162,6°W to 00,0°N/S 140,7°E

   ChLONG 56.7° ➢ 56.7°x60 = 3402 Nm

g. 49°N 178°E to 60°S 178°E

   ChLAT 109° ➢ 109°x60 = 6540 Nm

h. 0°N/S 179,5°E to 0°N/S 150,6°E

   ChLONG 28.9° ➢ 28.9°x60 = 1734 Nm

i. 0°N/S 171°30'E to 0°N/S 175°18'W

   ChLONG 13°12' ➢ 13°x60 +12' = 792 Nm

j. 78°N 120°E to 75°N 60°W

   (∑co-latitudes = 12°+15°) ChLAT 27° ➢ 27°x60 = 1620 Nm

Exercise 2

a. You travel West between A 0°N/S 32°30’W and B and cover 1093 Nm, what are the coordinates of B?
b. You travel East between C 0°N/S 140°42’E and D and cover 3050 Nm, what are the coordinates of D?
c. You travel North between E 32°30’N 132°30’E and F and cover 260 Nm, what are the coordinates of F?
d. You travel South between G 5°N 13°30’W and H and cover 370 Nm, what are the coordinates of H?

The correction is in the next page…

Exercise 2 (corrected)

a. You travel West between A 0°N/S 32°30’W and B and cover 1093 Nm, what are the coordinates of B?
b. You travel East between C 0°N/S 140°42’E and D and cover 3050 Nm, what are the coordinates of D?
c. You travel North between E 32°30’N 132°30’E and F and cover 260 Nm, what are the coordinates of F?
d. You travel South between G 5°N 13°30’W and H and cover 370 Nm, what are the coordinates of H?

a. **B (0°N/S 50°43’W)**

Explanation:

- You travel West, so you remain on the parallel 0°N/S, so B has the same latitude as A, so **0°N/S**
- ChLONG [A/B]:

1093 | 60
-60 | 18°
493
-480
13’

ChLONG [A/B]: 18°13’

B is 18°13’ west from A (32°30’W), so the longitude of B is **50°43’W**

b. 3050 Nm on Equator (Great Circle) → 50°50’
D (0°N/S 168°28’W)

c. 260 Nm on Meridian (Great Circle) → 4°20’
F (36°50’N 132°30’E)

d. 370 Nm on Meridian (Great Circle) → 6°10’
F (1°10’S 13°30’W)

***Tips*:**
If adding a ChLONG to a longitude value and the result is higher than 180° (crossing the Greenwich anti-meridian), simply subtract that number from 360° and assign to the result the designation East if the initial longitude is West and vice-versa to find the new longitude.

(eg. From 00°N/S 175°W with 20° ChLONG, 175 + 20 = 195 > 180°, 360° – 195° = 165°, so the result is 00°N/S 165°E)

Review

In GNAV, the Great Circle distances to be calculated are simply the distances on the ‘natural’ Great Circles:

1) Are the two points on the **same longitude** ?

- Yes, so it’s a Great Circle → ***Distance (Nm) = ChLAT (degree) x 60***
- No ↓

2) Are the two points **on the Equator**?

- Yes, so it’s a Great Circle → ***Distance (Nm) = ChLONG (degree) x 60***
- No ↓

3) Are the two points **on opposite meridians** (the sum of their longitudes is equal to 180°)?

- Yes, so it’s a Great Circle → ***Distance (Nm) = ∑co-latitudes (degree) x 60***
- No ↓

4) Are the two points on the **same latitude**?

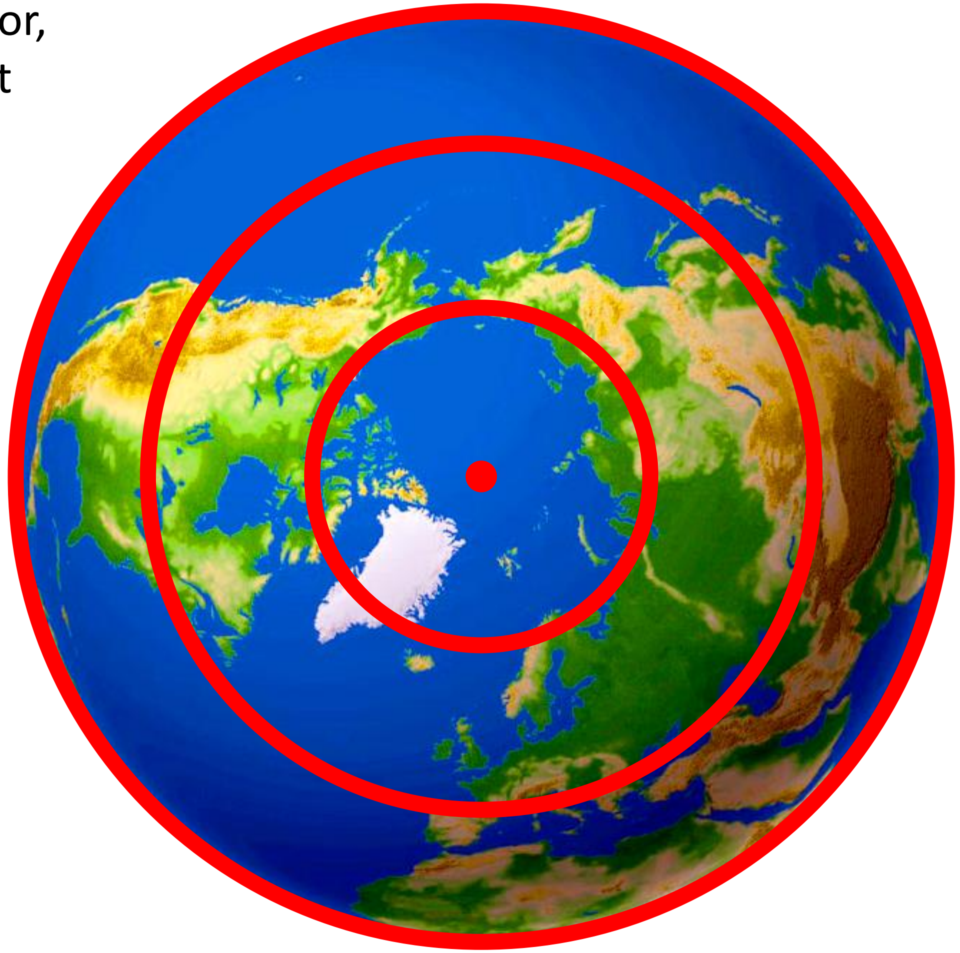
- Yes, this is not a Great Circle, however we can calculate the **Departure**.

Departure

The Departure is the distance on a parallel.

When two points are on the same latitude, other than the Equator, The distance between their change of longitudes will be different that if they were located at the Equator.

Indeed, since the parallels of latitudes have different radius, 1’ of arc is smaller than 1.852 km, and this is not 1 Nm since 1 Nm is 1’ of arc on the Great Circle. Notice that 1’ of arc, or 360° at the Pole will be 0 km.



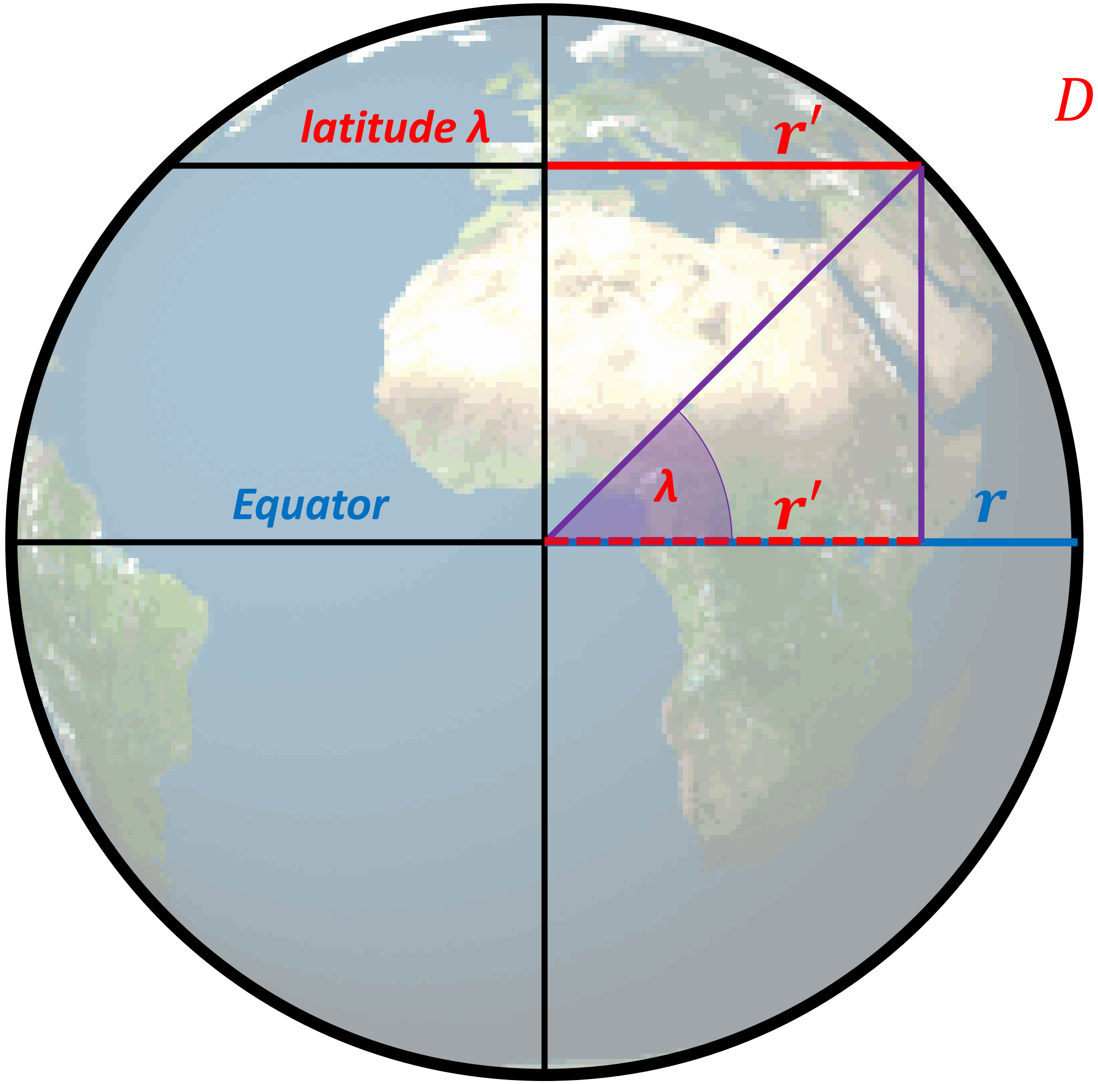
Departure calculation

To calculate a distance (**D**) at the **Equator**, we simply count the ChLONG (∡) because we know the value of the angle in distance since we know the radius (**r**).

We could calculate the distance (**D'**) for any **latitude λ** if we know the radius (**r'**) at that **latitude λ**.

To calculate the distance (**D'**), another method could be used, we could multiply the distance (**D**) at the **Equator** by the **ratio** between the radius (**r'**) at that **latitude λ** and the radius (**r**) at the **Equator**.

The ratio could be found by calculating the cosines of the angle between that **latitude λ** and the **Equator**, which is actually the **latitude λ**.



$$D' = \frac{2\pi r'}{360°} \text{ x } ∡$$

$$D' = D \text{ x } \frac{r'}{r}$$

$$D' = D \text{ x } \cos\lambda$$

$$D = \frac{2\pi r}{360°} \text{ x } ∡$$

Departure (Nm) = Distance at the Equator (Nm) x cos latitude λ

Departure (Nm) = ChLONG x 60 x cos latitude λ

Exercises

Departure (Nm) = ChLONG x 60 x cos latitude λ

a. What is the distance on the parallel between 45°N 075°W and 45°N 125°W?

b. What is the distance on the parallel between 60°S 001°20'E to 60°S 004°20'W?

c. What is the distance on the parallel between 30°42'N 171°30'E to 30°42'N 175°18'W?

d. A (60°N 032°30'W) is west of B by 1093 Nm, what are the coordinates of B?

e. C (30°S 140°42'E) is west of D by 3050 Nm, what are the coordinates of D?

The correction is on the next page…

Exercises *(corrected)*

Departure (Nm) = ChLONG x 60 x cos latitude λ

a. What is the distance on the parallel between 45°N 075°W and 45°N 125°W?

ChLONG 50° ➢ **Departure = 50° x 60 x cos 45° = 3000' x cos 45° = 2121 Nm**

b. What is the distance on the parallel between 60°S 001°20'E to 60°S 004°20'W?

ChLONG 5°40' ➢ **Departure = 5,666...° x 60 x cos 60° = 340' x cos 60° = 170 Nm**

c. What is the distance on the parallel between 30°42'N 171°30'E to 30°42'N 175°18'W?

ChLONG 13°12' ➢ **Departure = 13.2° x 60 x cos 30.7° = 792' x cos 60° = 681 Nm**

d. A (60°N 032°30'W) is west of B by 1093 Nm, what are the coordinates of B?

$$\text{ChLONG} = \frac{\text{Departure}}{60 \text{ x } \cos\lambda} = \frac{1093 \text{ Nm}}{60 \text{ x } \cos 60°} = 36.433...° = 36°26'$$

➢ **B (60°N 003°56'E)**

e. C (30°S 140°42'E) is west of D by 3050 Nm, what are the coordinates of D?

$$\text{ChLONG} = \frac{\text{Departure}}{60 \text{ x } \cos\lambda} = \frac{3050 \text{ Nm}}{60 \text{ x } \cos 30°} = 58.697...° \approx 58°42'$$

➢ **D (30°S 160°36'W)**

Map Projection

For the moment we will only discover how a map is made, but each type of map will be covered in details later.

We sometimes say chart instead of map, the difference is, a map has a lot of details and a chart only the most important information.

We speak about map projection because it consist of lightening the globe, and draw the projected shadow of the areas on a paper.

There is a large number of different type of projections, however in this lesson, we will cover only 3 types of projections, and mention few different technics:

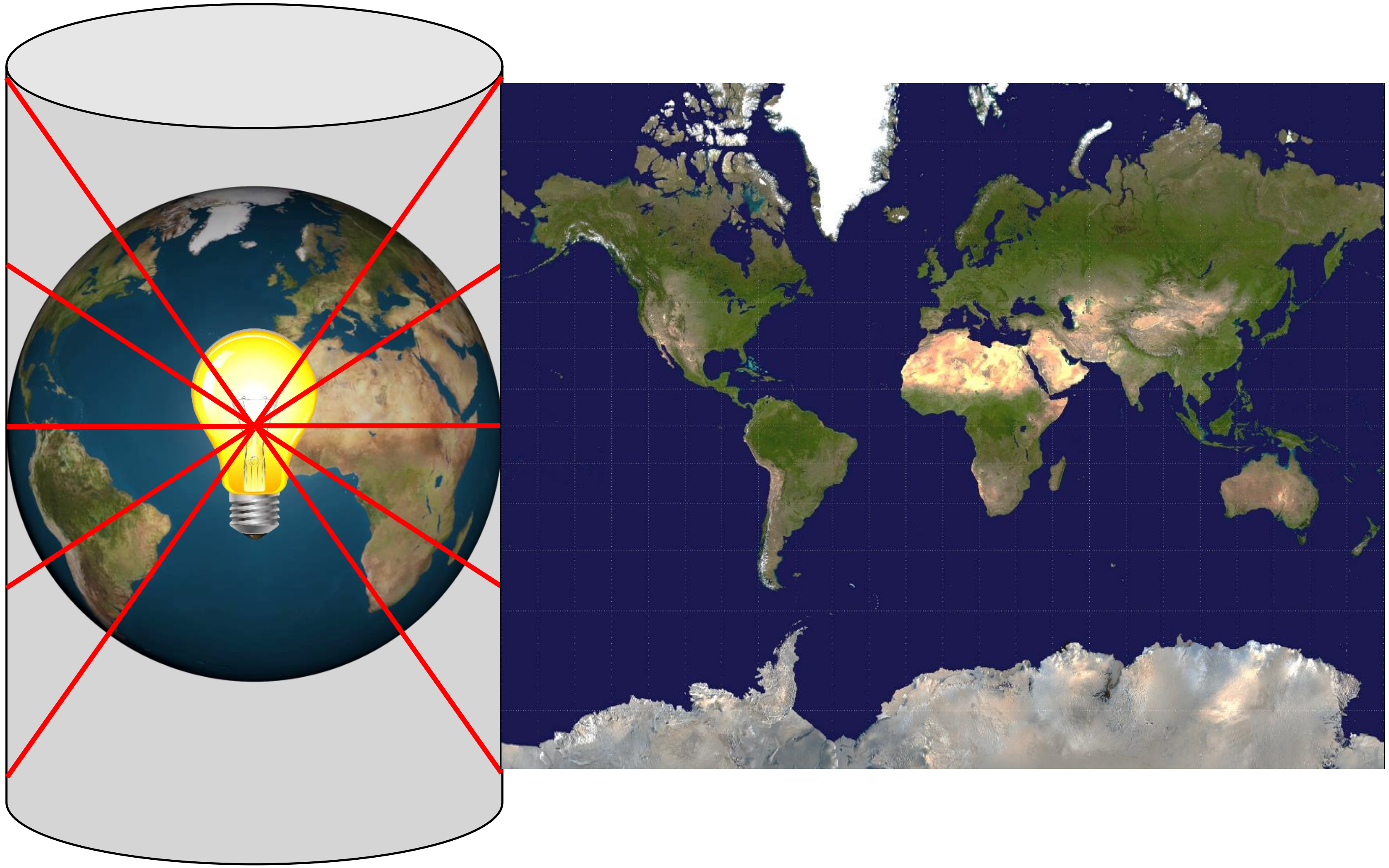
- Mercator (Direct, Oblique and Transverse)
- Lambert Conical (Simple and Orthomorphic)
- Polar (Stereographic)

Mercator projection

It is a cylindrical map projection presented by the Flemish geographer and cartographer Gerardus Mercator (1512-1594) in 1569.

Gerardus Mercator (1512–1594)

Mercator projection

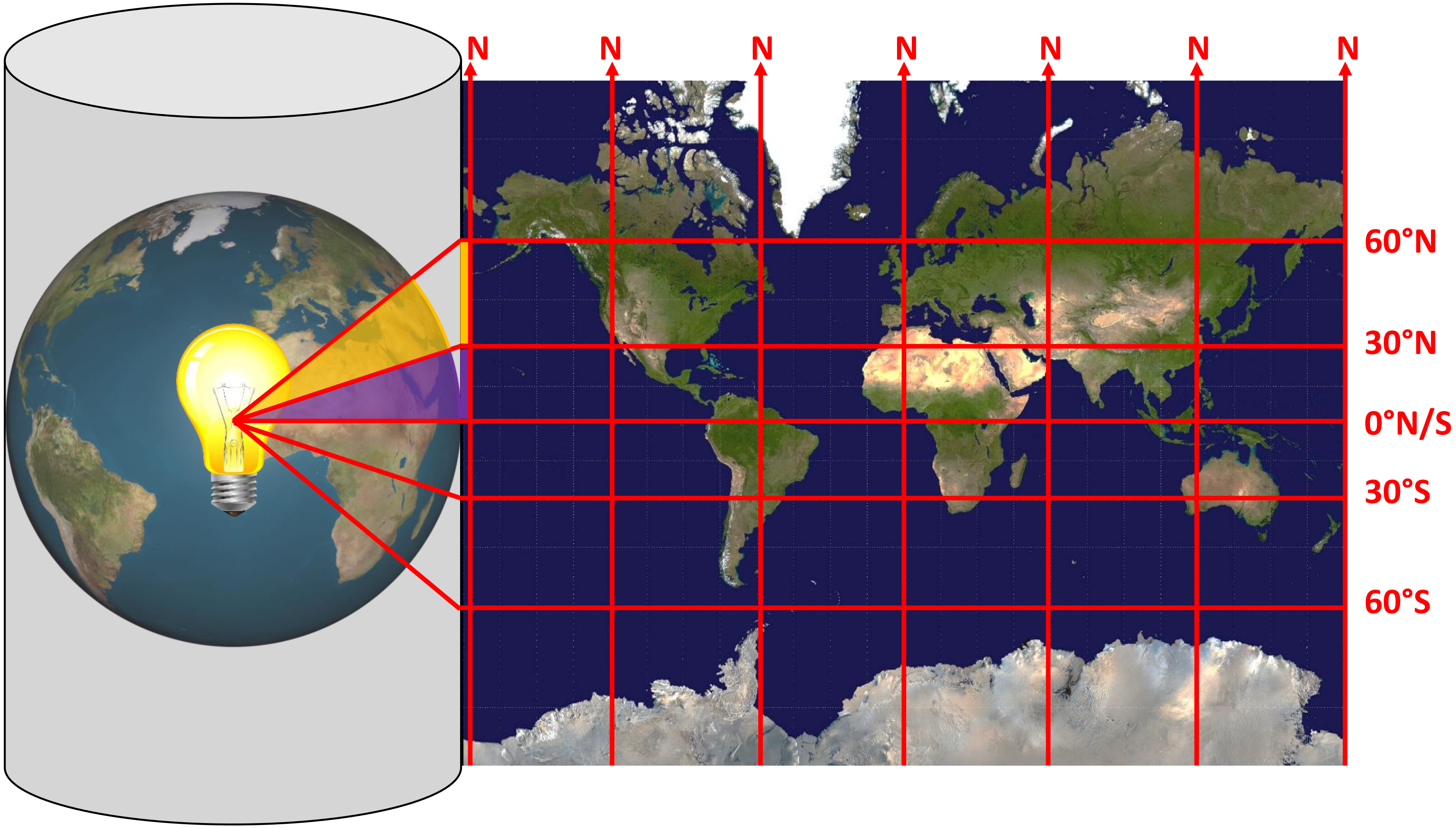


The earliest method of transferring the graticule of meridians and parallels from a globe to a flat sheet of paper was achieved using cylindrical projections. A scale model of the earth, the Reduced Earth (RE), was made at an appropriate scale. A cylinder of paper was wrapped around the RE, touching the RE at the Equator. Using a light source at the centre of the RE, the graticule was projected onto the cylinder. The cylinder was then 'developed' or opened up to a flat sheet of paper.

As it can be seen, some problems exist on the map. At the Equator and around, the areas shape a correct, however, away from the Equator, the area and the graticule are expanding (Alaska is larger than Africa).

This is because the cylinder is tangent to the Equator and the shadows will expands when becoming further from the Equator.

This map is used for the Equatorial region since the area scale is correctly projected there.



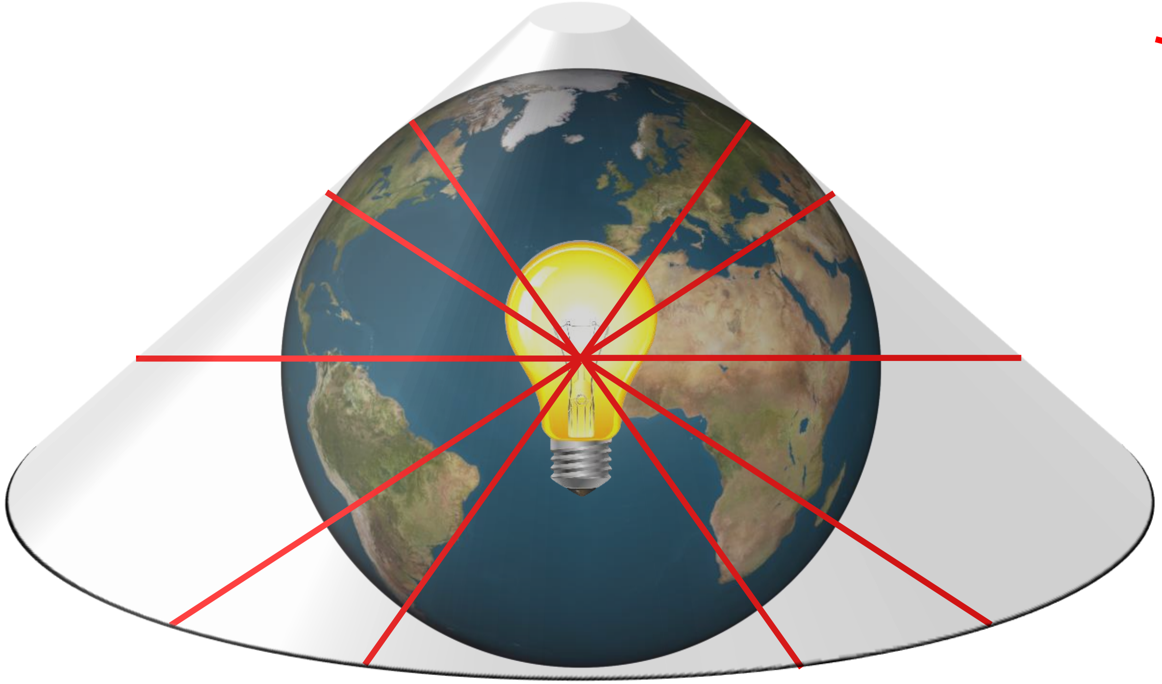
Notice that the meridians are parallel and the North is everywhere at the top of each meridian (which makes it impossible to navigate at the poles with this projection.

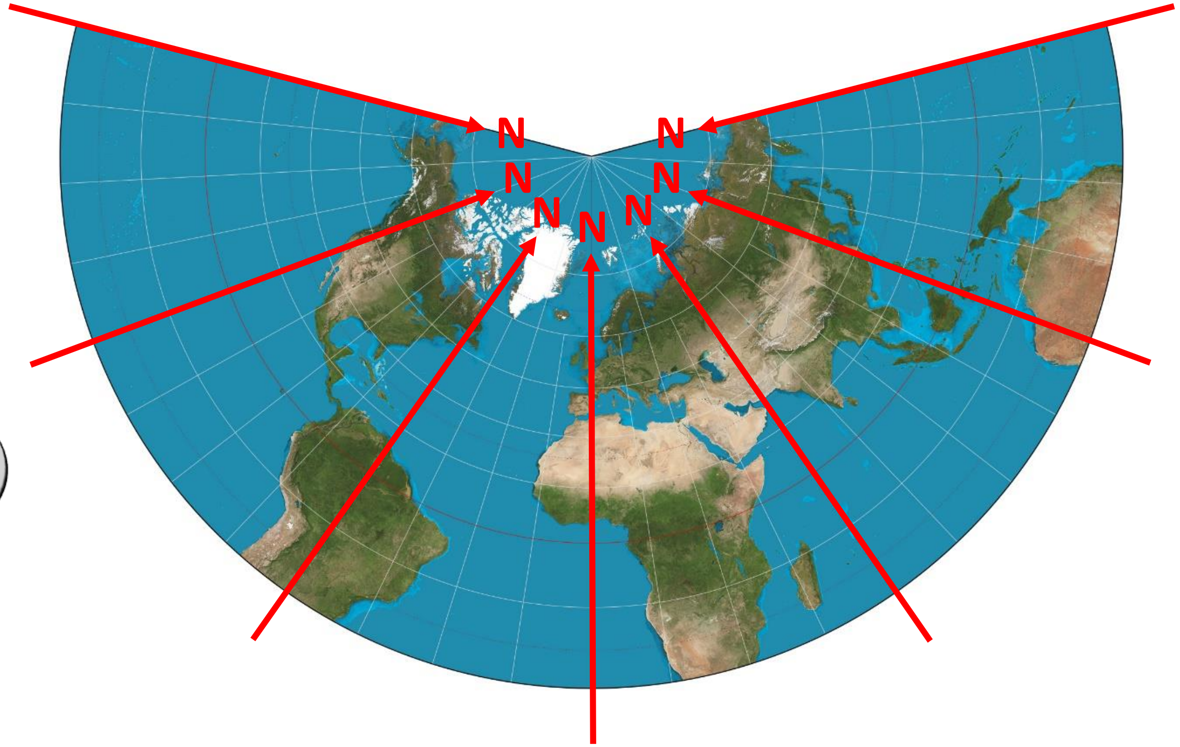
Lambert conformal conic projection (LCC)

It is a conic map projection used for aeronautical charts, is one of seven projections introduced by the Swiss mathematician and philosopher Johann Heinrich Lambert (1728-1777) in his 1772 publication.

Johann Heinrich Lambert (1728–1777)

Lambert conformal conic projection (LCC)





A cone is placed over a reduced earth, in such a way that the cone is tangential with the reduced earth along a parallel of latitude.

On this map, since the paper in tangent to the mid latitude, on that latitude the shape is correct, however further away, the graticule and the areas are expanding, making this map only good to be used at the mid-latitudes.

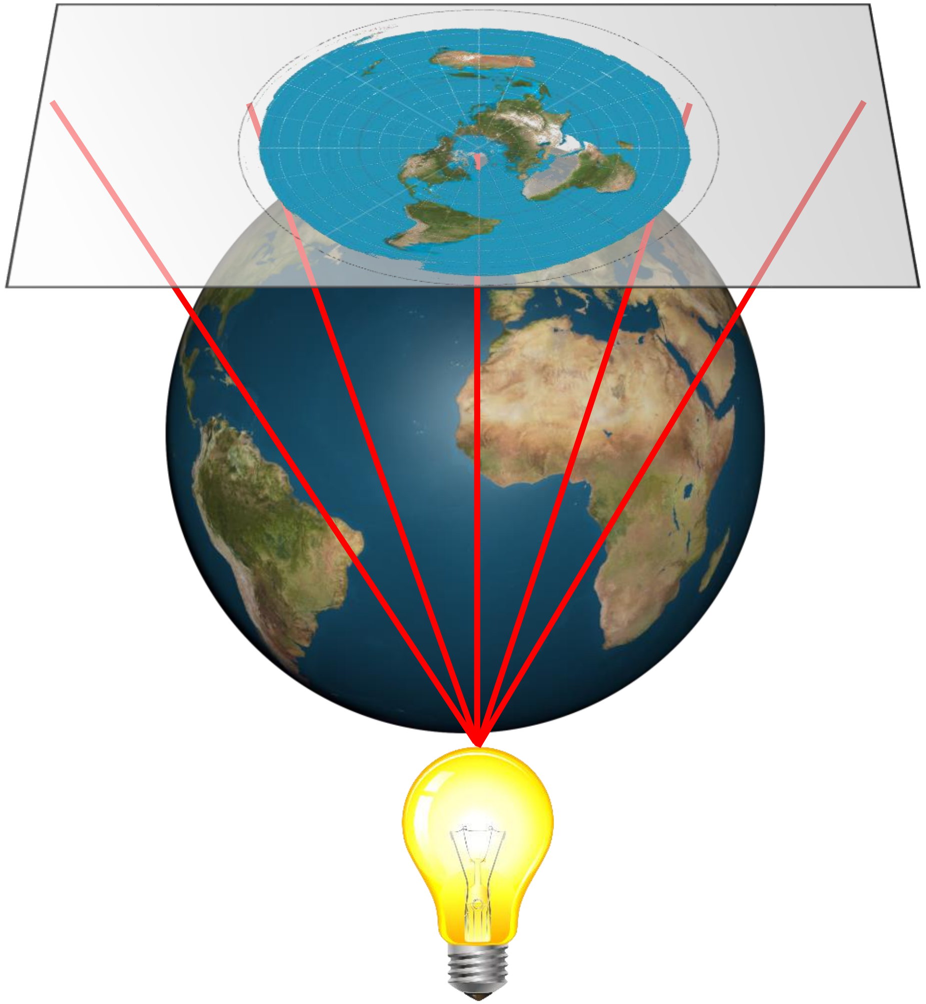
Notice that the meridians are straight lines, originating from the pole and they have the same North.

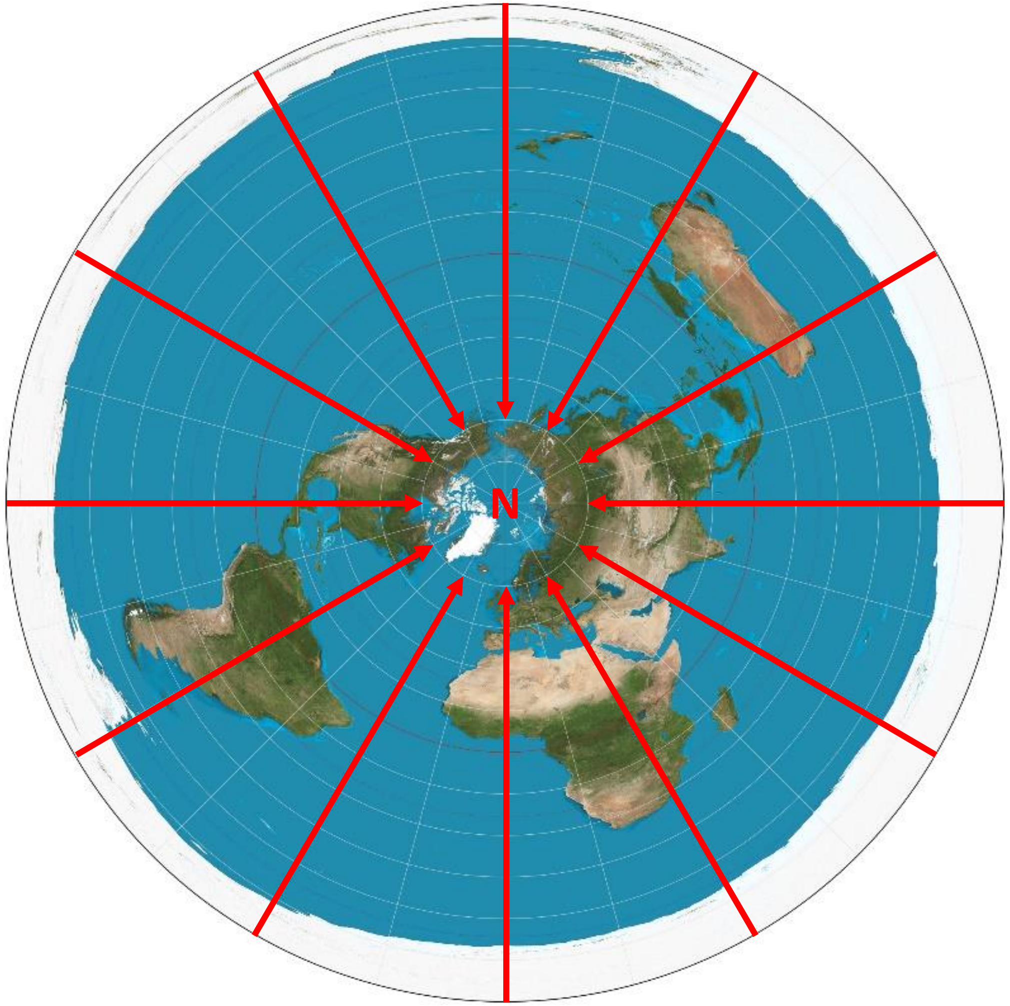
Polar Stereographic projection

While it may have been used by ancient Egyptians for star maps in some holy books, the earliest text describing the azimuthal equidistant projection is an 11th-century work by the Persian mathematician and astronomer Abu Rayhan Al-Biruni (973-1048).

Abu Rayhan Al-Biruni (973–1048)

Polar Stereographic projection



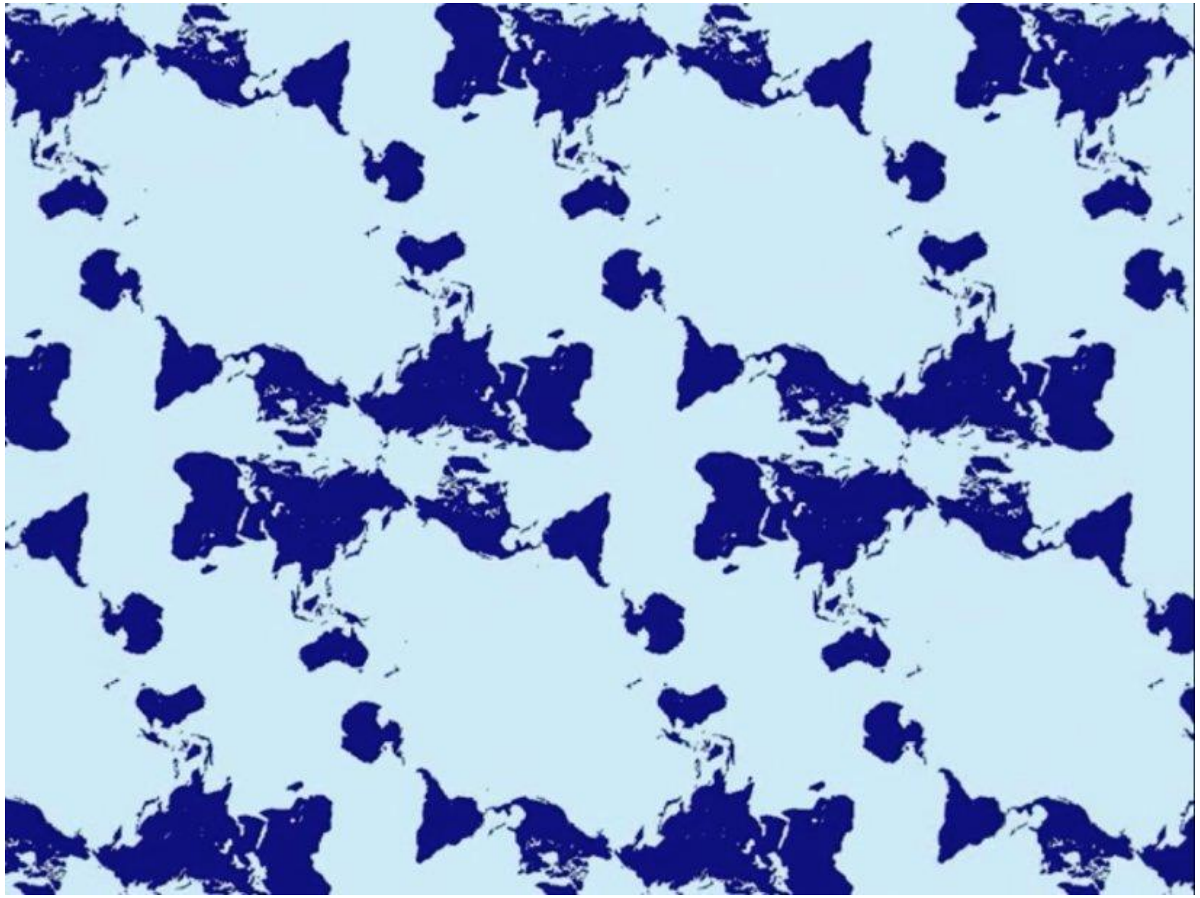


A flat surface is used, touching the North Pole (the point of tangency). The light source is positioned at the South Pole (diametrically opposed), creating a graticule, by geometrical projection, which is shown in the lower part of the diagram.
Since the surface to perpendicular to the pole, the polar area will be correctly projected which could be used for polar navigation.
Notice that Meridians are straight lines radiating from the Pole and the North is one point.

We have seen that, any Each projection distorts the Earth and indeed, there is no projection that could display the Earth without distortion, this is because it is mathematically impossible to convert a 3D sphere into a 2D plane.

When attempting to resolve this mathematical conversion from 3D sphere into 2D plane, the result is infinite.
In fact, this result isn't meaningless, indeed if we project every single point of the Earth's surface tangent to the flat sheet then put all the sheets together like a 'puzzle', the result is a an infinite map, where the areas keep repeating.

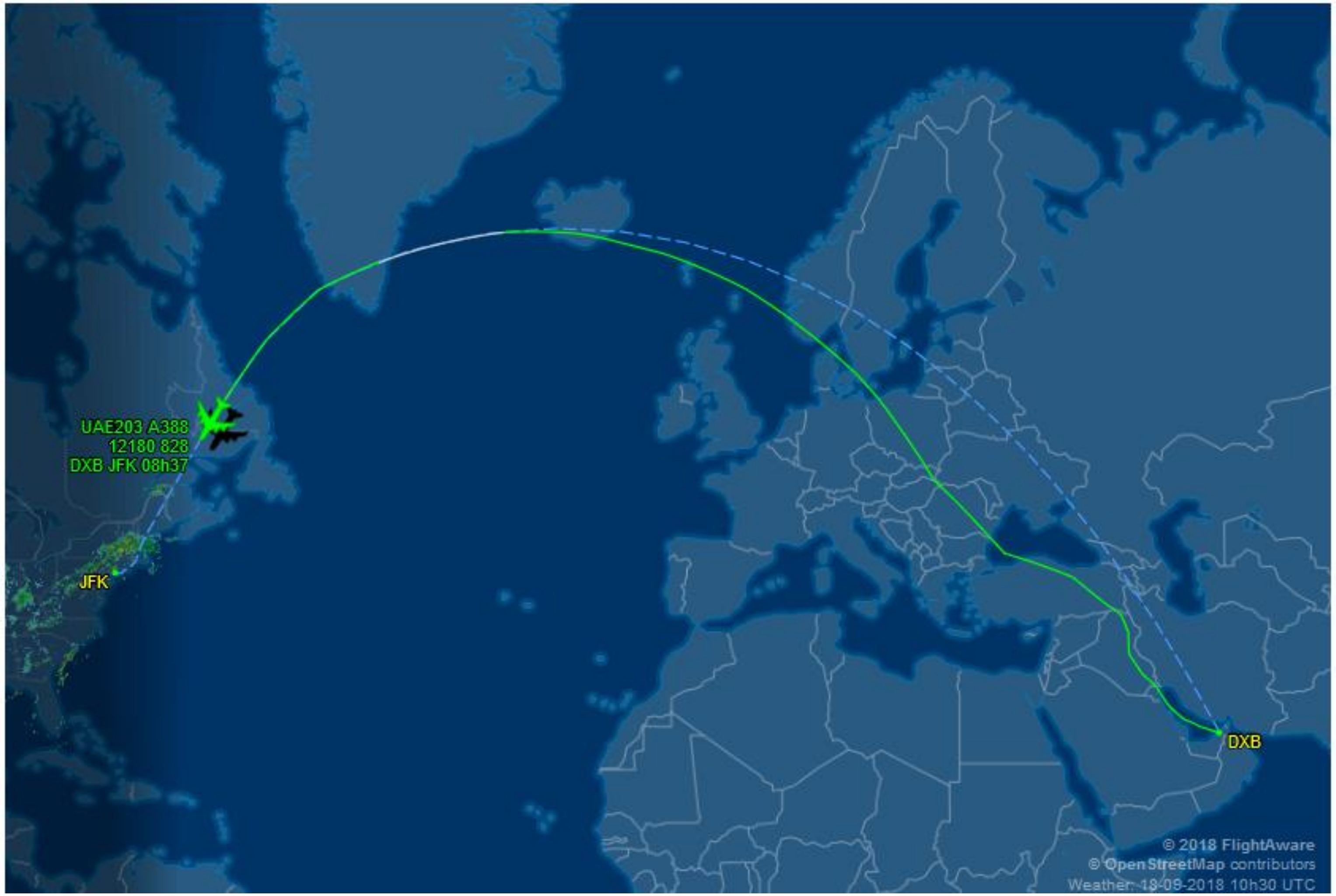
However this map isn't representative and useless for navigation purpose.



This was only an introduction to the types of map projection to understand the next part.

Which map shall we use?

Let's have a look to a scheduled flight on a flight radar: Emirates EK203 from Dubai to New York JFK.

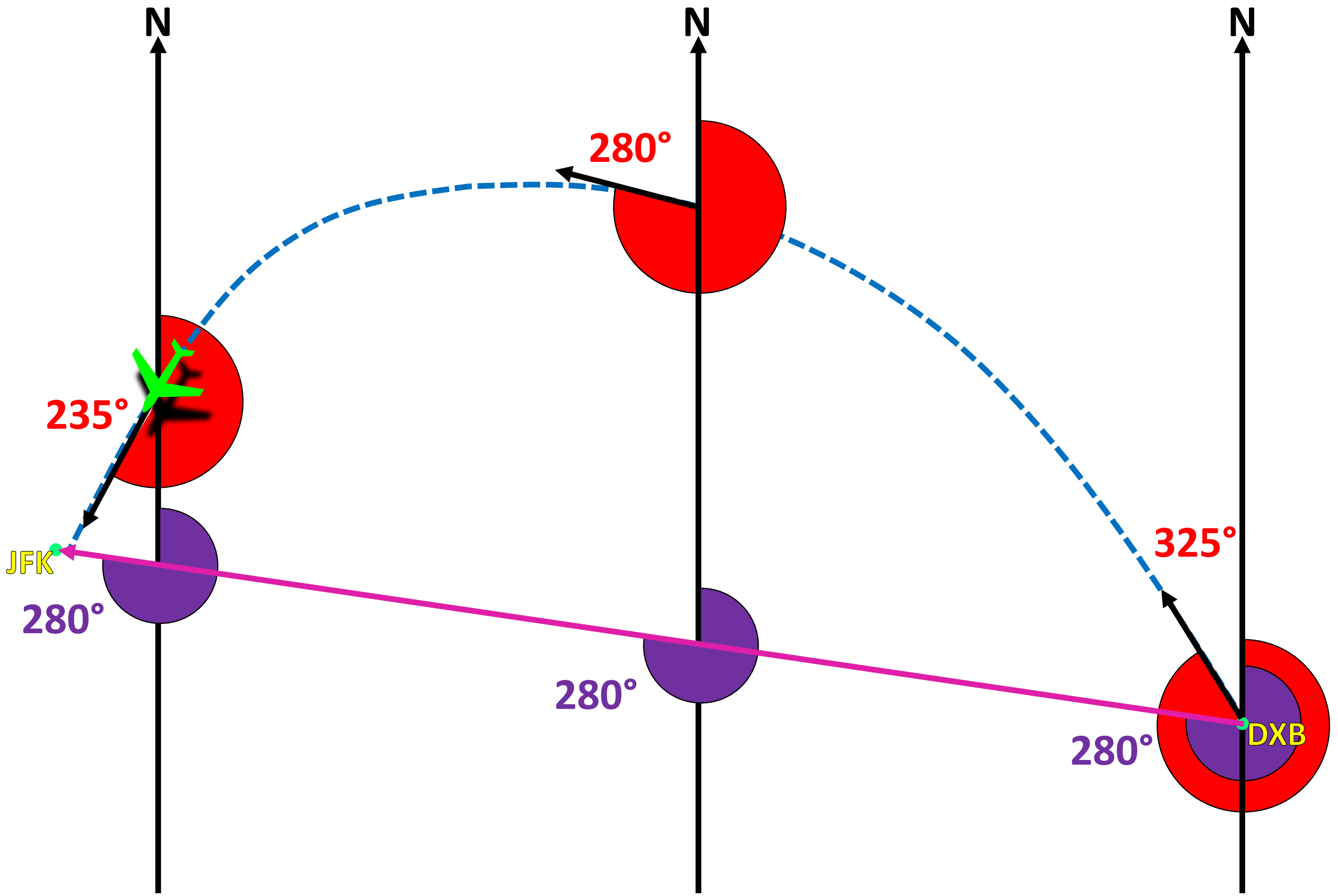


On the dashed blue line, we see that the scheduled track is via Iran, the Baltic countries, Iceland, south of Greenland and finally via North-East Canada.

If we analyse the track on this Mercator map by placing some meridians:

The track is initially North-Northwest (≈325°), to become nearly West (≈280°) halfway, and end up South West (≈235°).

This trajectory might look strange as it doesn't seem to be the shortest distance between DXB and JFK.

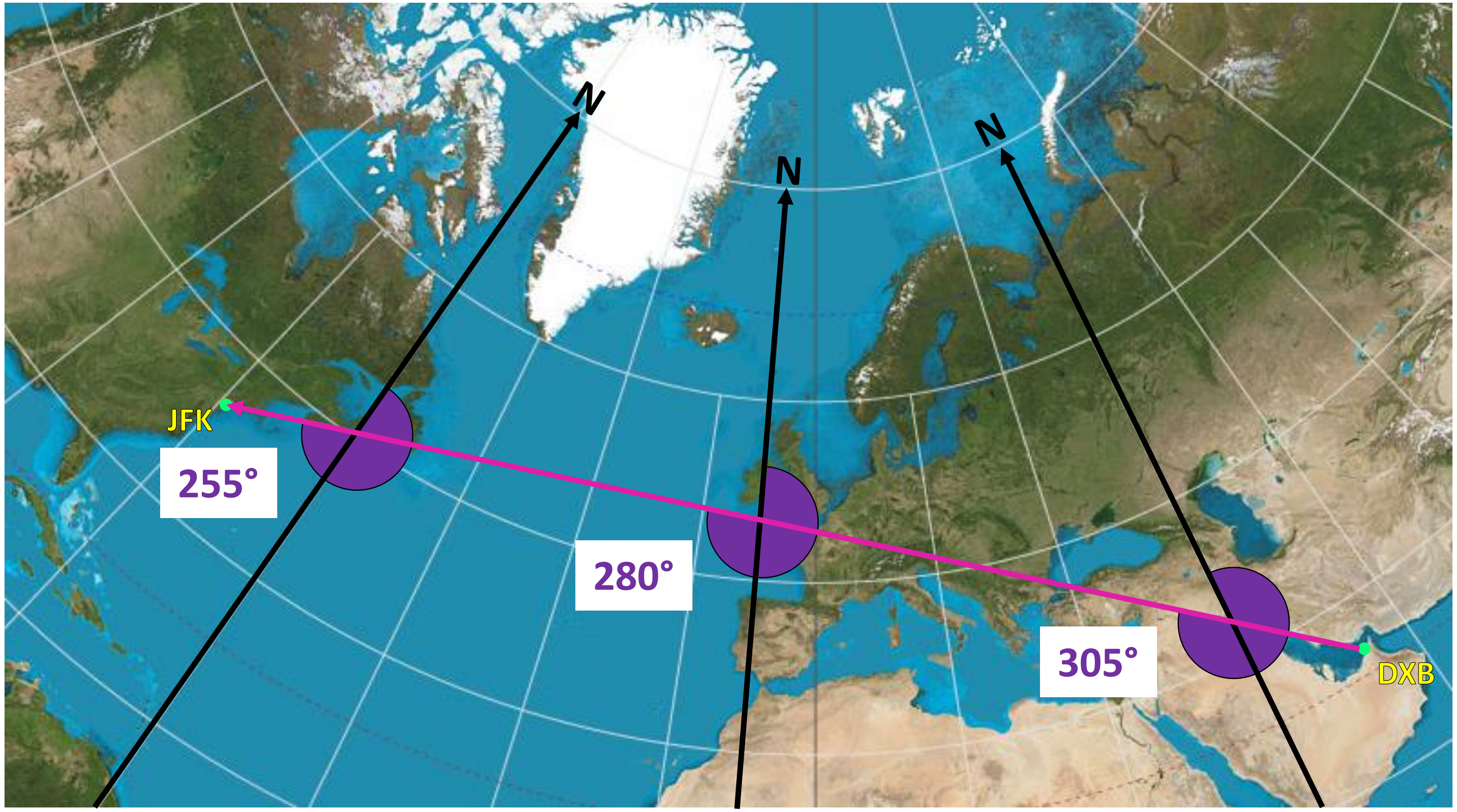


However, if we ask any idiot to draw the shortest distance on the **Mercator map** , he/she will obviously draw a straight line.

This line is crossing all the meridians at the same angle, it's a Rhumb Line, and according to the Rhumb Line, the shortest track is to keep flying ≈280° via North Africa

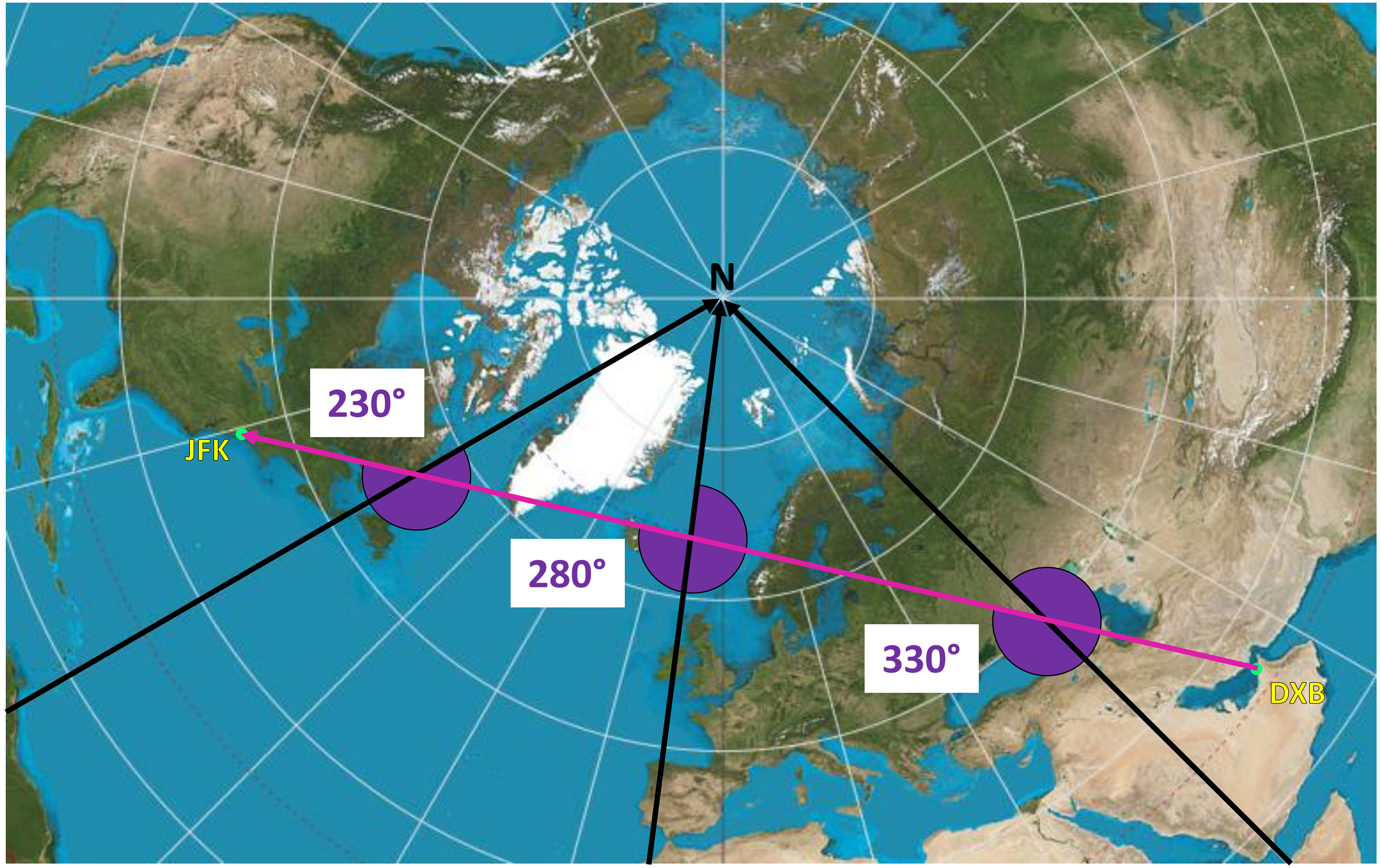
If we try to draw the shortest distance between DXB and JFK on a **Lambert Conformal Map projected at 45°N**:

The track is initially North-Northwest (≈305°), to become nearly West (≈280°) halfway, and end up West South West (≈255°).



If we try to draw the shortest distance between DXB and JFK on a **Polar Streographic Map projected at 90°N**:

The track is initially North-Northwest (≈330°), to become nearly West (≈280°) halfway, and end up South West (≈230°).



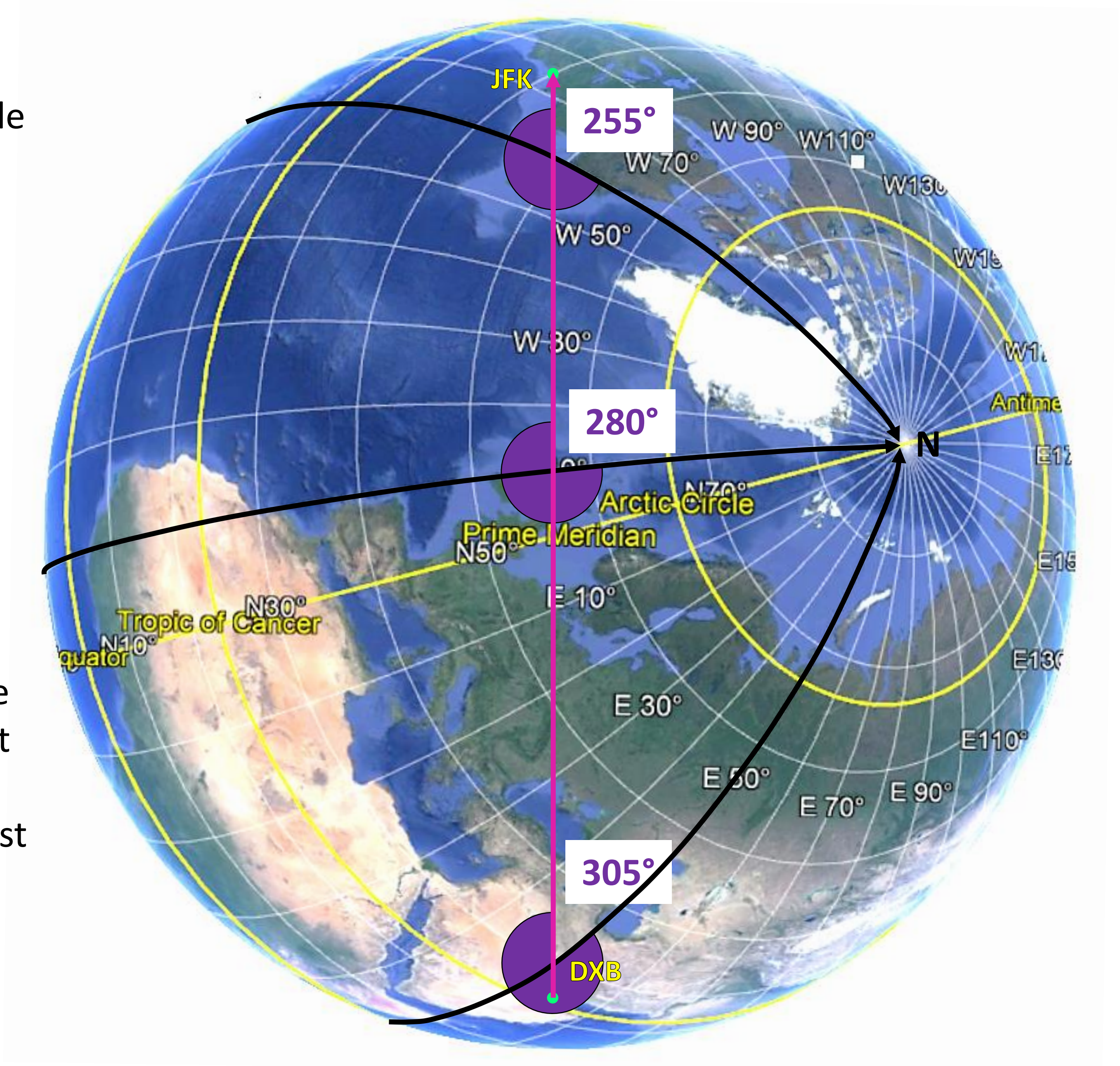
Every map propose a different ‘shortest’ route to fly navigate from DXB to JFK. The question is: which one shall we use?

Notice that on the 3 maps, the halfway track is the same, which is also the average track between the initial and the final track.

View	Initial Track	Halfway Track	Final Track
Flight Radar path	325°	280°	235°
Mercator straight line	280°	280°	280°
Lambert straight line	305°	280°	255°
Polar straight line	330°	280°	230°

To verify which projection is appropriate to navigate between DXB and JFK, we will compare the maps with the shortest route on the real Earth.

we need to draw the Great Circle between DXB and JFK since it is the shortest distance on Earth between 2 points.



We can see that The Great Circle track is initially North-Northwest (≈305°), to become nearly West (≈280°) halfway, and end up West South West (≈255°).

Let’s add these observed track angle of the Earth Great Circle to the table to compare it with the previous straight lines track on the maps:

When we compare the track angles, we notice a straight line drawn between DXB and JFK on Lambert map has the same track angles as the Great Circle track angles on Earth.

Indeed DXB (25°15’N) and JFK (40°38’N) are mid-latitudes location (33°N average latitude), and the Lambert map is designed for mid-latitudes. This means that navigating from DXB to JFK with a Lambert map projected at 33°N latitude, will allow you to follow a Great Circle, so the shortest distance between these to points.

The map chosen to navigate between two points, is the one that correctly projects the areas between these points, as well the one that allows you to draw and follow a straight line which matches the Great Circle on Earth.

View	Initial Track	Halfway Track	Final Track
Flight Radar path	325°	280°	235°
Mercator straight line	280°	280°	280°
Lambert straight line	**305°**	**280°**	**255°**
Polar straight line	330°	280°	230°
Earth Great Circle	**305°**	**280°**	**255°**