# **MASS & BALANCE ATPL AB-INITIO**

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# I) INTRODUCTION

In this lesson, the weight of the aircraft will be studied in order to understand how to manage it. The management of the aircraft's weight is subject to many limitations, that in case of non-compliance of them, the flight safety is jeopardised.

First of all, the terms "weight" and "mass" are generally confused and misunderstood. Mass (m) can be generalized as the amount of matter in an object, whereas the Weight (W) is of an object is related to the amount of force acting on the object due to gravity (W=m.g), the lift generated by the wings, is the force that will act against the aircraft's weight. In this lesson, the term "mass" will be used.

The aircraft, as any structure, has a maximum structural mass limitation that could be set by the maximum strength of its floors, the airframe strength to resist to the external forces existing in different phases of the flight, or the performances of the aircraft in different phases of the flight which are limited by the maximum lift capability of the wings.

Exceeding the published limitations of the aircraft may lead to incidents or accidents, due to:

- Deteriorated aircraft's performances
- Permanent structural deformation
- Structural failures

Other limitation exist to define the location of the Centre of Gravity of the aircraft, the point where the total weight is acting, in order to maintain an acceptable stability and manoeuvrability of the aircraft. **The Centre of Gravity (CG) is:** 

- the point that the total weight of the aircraft is said to act through
- the point of balance
- that part of the aircraft that follows the flight path
- the point that the aircraft manoeuvres about in the air
- the point that the three axes of the aircraft pass through.

When the CG is forward (FWD), the lift generated from the wings will generate a greater nose down moment to the aircraft. In order to maintain the aircraft stable, the tailplane must deflected up to generate a vertical downward force to keep the nose up. The upward deflection of the tailplane will make it less streamlined into the airflow resulting in an increase of the total drag, that will require more thrust from the engine to be compensated, meaning that the fuel flow (fuel consumption) increases, leading to a decrease of the aircraft maximum endurance and range. Furthermore, the vertical downward force generated by the tailplane will be added to the total weight of the aircraft and make it effectively heavier, thus resulting in an increase of the stall speed and other performances speeds. The total opposite is true when CG is afterward (AFT).

When the CG is forward (FWD), since the aircraft has nose down tendency, this will make it more stable along its longitudinal axis, in the event of a disturbance from an upward gust. Increasing the aircraft longitudinal stability causes a decrease in its longitudinal manoeuvrability. The total opposite is true when the CG is afterward (AFT)



	AFT CG	FWD CG
Effective aircraft weight	$\mathbf{\bullet}$	<b>^</b>
Total Drag	$\checkmark$	<b>^</b>
$V_s$ and $V_R$	$\checkmark$	<b>^</b>
Endurance	<b>^</b>	V
Range	1	V
Longitudinal static stability	$\checkmark$	<b>^</b>
Longitudinal static manoeuvrability	<b>^</b>	•

The CG position must be find within defined limit established by the manufacturer of the aeroplane. The FWD CG limit will be the most FWD allowable CG position and will be determined according to the maximum upward stabiliser/elevator deflection, as well according to minimum mean acceptable manoeuvrability to maintain a positive and safe aeroplane control. The AFT CG limit will be the most AFT allowable CG position and will be determined according to the maximum downward stabiliser/elevator deflection, as well according to minimum mean acceptable stabiliser/elevator deflection, as well according to the maximum downward stabiliser/elevator deflection, as well according to minimum mean acceptable stability to maintain a positive and safe aeroplane control.

In flight, the mass of the aircraft will decrease and the CG in most of the cases shift, therefore the commander must ensure that the CG remains within the allowed limit during the entire trip.

When the FWD CG limit is exceeded:

- Difficulties to pitch up or down
- Difficulties to rotate the aeroplane during take-off
- Difficulties to flare the aeroplane during landing

When the AFT CG limit is exceeded:

- Difficulties to maintain a positive control due to increased manoeuvrability
- Stick force is reduced and the pilot may overstress the aeroplane
- Uncontrollable early rotate during take-off

In this lesson it will be seen how the CG position is calculated and adjusted, how the aircraft is kept within the allowable mass limitations before operation.

# II) **DEFINITIONS**

# **A- MASS DEFINITION**

**Basic Empty Mass (BEM):** is the mass of an aeroplane plus standard items such as:

- Unusable fuel and other unusable fluids;
- Lubricating oil in engine and auxiliary units;
- Fire extinguishers;
- pyrotechnics;
- emergency oxygen equipment;
- Supplementary electronic equipment.

**Dry Operating Mass (DOM)**: is the total mass of the aeroplane ready for a specific type of operation excluding usable fuel and traffic load. The mass, sometimes called the **Variable Load (VL)**, includes items such as:

- Crew and crew baggage.
- Catering and removable passenger service equipment.
- Potable water and lavatory chemicals.
- Food and beverages.

# DOM = BEM + Crew and their baggage & Service Equipment DOM = BEM + VL

**Traffic Load (TL)**: also known as **payload**, is the total mass of passengers, baggage and cargo, including any 'non-revenue' load.

Take-Off Fuel (TOF): Fuel on-board prior to take-off

**Zero Fuel Mass (ZFM):** is DOM plus traffic load but excluding fuel. The ZFM must not exceed the Maximum Zero Fuel Mass

# ZFM = DOM + TL

Maximum Zero Fuel Mass (MZFM): is the maximum permissible mass of an aeroplane with no usable fuel.

The MZFM is definitely a fixed limit and is calculated on the maximum permissible bending moment at the wing root. Maximum load factor at the MZFM is + 2.5 g under CS25.

**Operating Mass (OM)**: is the DOM plus Take-Off fuel (TOF) but without traffic load.

**Useful Load (UL)**: is the total mass of the passengers, baggage and cargo, including any non-revenue load and usable fuel.

#### UL = TOF + TL

**Taxi Mass** or **Ramp Mass**: is the mass of the aeroplane at the start of the taxi (at departure from the loading gate).

#### Ramp Mass = DOM + TL + Block Fuel

**Maximum Structural Taxi Mass**: is the structural limitation of the mass of the aeroplane at commencement of taxi.

The Taxi Mass must not exceed the Maximum Structural Mass

**Start and Taxi Fuel**: Fuel required for an aeroplane to start and taxi from the gate (ramp) to the runway to start the take-off

Block Fuel: The mass of the total usable fuel on-board at the ramp

#### Block Fuel = Taxi Fuel + TOF

**Take-Off Mass (TOM)**: is the mass of the aeroplane including everything and everyone contained within it at the start of the take-off run.

# TOM = DOM + UL TOM = ZFM + TOF TOM = OM + TL TOM = Ramp Mass - Start and Taxi Fuel

**Maximum Structural Take-Off Mass (MSTOM)**: the maximum permissible total aeroplane mass at the start of the take-off run.

**Performance Limited Take-Off Mass (PLTOM):** is the take-off mass subject to departure aerodrome limitations.

**Regulated Take-Off Mass (RTOM):** is the lowest of the 'performance limited' TOM and 'structural limited' TOM. The TOM must not exceed the RTOM **Trip Fuel:** The Trip fuel is the required fuel quantity from brake release on takeoff at the departure aerodrome to the landing touchdown at the destination aerodrome. This quantity includes the fuel required for:

- Take-off
- Climb to cruise level
- Flight in level cruise including any planned step climb or step descent
- Flight from the beginning of descent to the beginning of approach,
- Approach
- Landing at the destination

Trip fuel must be adjusted to account for any additional fuel that would be required for known ATS restrictions that would result in delayed climb to or early descent from planned cruising altitude.

# Landing Mass (LM):

## LM = TOM – TRIP FUEL

**Maximum Structural Landing Mass (MSLM):** the maximum permissible total aeroplane mass on landing in normal circumstances.

**Performance Limited Landing Mass (PLLM)**: is the mass subject to the landing aerodrome limitations.

**Regulated Landing Mass (RLM):** is the lowest of the 'performance limited' landing mass and 'structural limited' landing mass. The LM must not exceed the RLM

**Underload:** The mass of the traffic load that can be added to the actual traffic load without exceed the Allowable Traffic Load in case of a Last Minute Change (LMC) before departure

## Underload = Allowable TL – Actual TL

All-Up Mass (AUM): The mass of an aircraft is known as the all-up mass (AUM), gross mass or sometimes total mass

## AUM = BEM + VL + TL + Fuel



**Allowable Traffic Load:** The maximum Traffic Load that can be loaded without exceeding the maximum limitations of the aircraft.

It has been seen that the TOM will be must not exceed The RTOM:

Normally, the commander will first defined the required fuel, then he/she will find the Allowable Traffic Load (TL) to not exceed the RTOM

#### So the maximum allowable TL is <u>limited by the RTOM</u>, i.e.:

RTOM: 100 000 kg DOM: 20 000 kg TOF: 55 000 kg Allowable TL = RTOM – OM = RTOM – (DOM + TOF) = 100 000 kg – (20 000 kg + 55 000 kg) Allowable TL = 25 000 kg

#### However, the TOM might me be <u>limited by the (MZFM + TOF)</u>, i.e.:

RTOM: 100 000 kg DOM: 20 000 kg MZFM: 55 000 kg TOF: 37 000 kg Allowable TL = RTOM – OM = RTOM – (DOM + TOF) = 100 000 kg – (20 000 kg + 37 000 kg) Allowable TL = 43 000 kg

However, if the commander load 43 000 kg of TL the MZFM is exceeded: ZFM = DOM + TL = 20 000 kg + 43 000 kg = 63 000, the MZFM is exceeded by 8 000 kg.

If we look at the difference between the DOM and the MZFM (DOM – MZFM), we can see that the actual allowable TL is  $35\ 000$  kg.

To avoid such error to occur, the TOM will be limited by the MZFM: Allowable TL =  $(MZFM + TOF) - OM = (55\ 000\ kg + 37\ 000\ kg) - 57\ 000\ kg = 35\ 000\ kg$ 

To define the Maximum Allowable TL, the commander must set the Maximum TOM according to the lesser of the RTOM and (MZFM + OM)

As well, the TOM might be <u>limited by the (RLM + Trip Fuel)</u>, i.e.:

RTOM: 100 000 kg DOM: 20 000 kg TOF: 35 000 kg RLM: 75 000 kg Trip Fuel: 22 000 kg Allowable TL = RTOM – OM = RTOM – (DOM + TOF) = 100 000 kg – (20 000 kg + 35 000 kg) Allowable TL = 45 000 kg

However, if the commander load 45 000 kg of TL when he/she lands after consuming the planned trip fuel, the RLM will be exceeded: LM = RTOM – Trip Fuel = 100 000 kg – 22 000 kg = 78 000 kg, the RLF is exceeded by 3 000 kg

To avoid such error to occur, the TOM will be limited by the RLM: Allowable TL =  $(RLM + Trip Fuel) - OM = (75\ 000\ kg + 22\ 000\ kg) - 55\ 000\ kg = 42\ 000\ kg$ 

To define the Maximum Allowable TL, the commander must set the Maximum TOM according to the lesser of the RTOM, (MZFM + OM) and (RLM + Trip Fuel).

#### SUMMARY:

To determine the Maximum Allowable TL, the commander must subtract the OM from the MTOM (MTOM – OM). The MTOM is the lesser of:

- RTOM
- MZFM + TOF
- RLM + Trip Fuel

#### **B- REGULATION**

# According to Regulation (EU) 965/2012 on air operations Annex IV – Part-CAT version March 2018

(!) To know for the questions/or most relevant information for the lesson

# 1- Introduction

## According to CAT.POL.MAB.100 Mass and balance, loading

(a) During any phase of operation, the loading, mass and centre of gravity (CG) of the aircraft shall comply with the limitations specified in the AFM, or the operations manual if more restrictive.

(b) The operator shall establish the mass and the CG of any aircraft by actual weighing prior to initial entry into service and thereafter at intervals of four years if individual aircraft masses are used, or nine years if fleet masses are used. The accumulated effects of modifications and repairs on the mass and balance shall be accounted for and properly documented. Aircraft shall be reweighed if the effect of modifications on the mass and balance is not accurately known.

(c) The weighing shall be accomplished by the manufacturer of the aircraft or by an approved maintenance organisation.

(d) The operator shall determine the mass of all operating items and crew members included in the aircraft dry operating mass by weighing or by using standard masses. The influence of their position on the aircraft's CG shall be determined.

(e) The operator shall establish the mass of the traffic load, including any ballast, by actual weighing or by determining the mass of the traffic load in accordance with standard passenger and baggage masses.

(f) In addition to standard masses for passengers and checked baggage, the operator can use standard masses for other load items, if it demonstrates to the competent authority that these items have the same mass or that their masses are within specified tolerances.

(g) The operator shall determine the mass of the fuel load by using the actual density or, if not known, the density calculated in accordance with a method specified in the operations manual.

(h) The operator shall ensure that the loading of:

(1) its aircraft is performed under the supervision of qualified personnel; and

(2) traffic load is consistent with the data used for the calculation of the aircraft mass and balance.

(i) The operator shall comply with additional structural limits such as the floor strength limitations, the maximum load per running metre, the maximum mass per cargo compartment and the maximum seating limit. For helicopters, in addition, the operator shall take account of in-flight changes in loading.

(j) The operator shall specify, in the operations manual, the principles and methods involved in the loading and in the mass and balance system that meet the requirements contained in (a) to (i).

This system shall cover all types of intended operations.

# 2- FLEET MASS AND CG POSITION — AEROPLANES According to AMC2 CAT.POL.MAB.100(b) Mass and balance, loading

(a) For a group of aeroplanes of the same model and configuration, an average dry operating mass and CG position may be used as the fleet mass and CG position, provided that:

(1) the dry operating mass of an individual aeroplane does not differ by more than  $\pm 0.5\%$  of the maximum structural landing mass from the established dry operating fleet mass; or

(2) the CG position of an individual aeroplane does not differ by more than  $\pm 0.5\%$  of the mean aerodynamic chord from the established fleet CG.

(b) The operator should verify that, after an equipment or configuration change or after weighing, the aeroplane falls within the tolerances above.

(c) To add an aeroplane to a fleet operated with fleet values, the operator should verify by weighing or calculation that its actual values fall within the tolerances specified in (a)(1) and (2).

(d) To obtain fleet values, the operator should weigh, in the period between two fleet mass evaluations, a certain number of aeroplanes as specified in Table 1, where 'n' is the number of aeroplanes in the fleet using fleet values. Those aeroplanes in the fleet that have not been weighed for the longest time should be selected first.

### (!) Table 1

Minimum number of weightings to obtain fleet values

Number of aeroplanes in the fleet (n)	Minimum number of weighting
2 or 3	n
4 to 9	(n+3)/2
10 or more	(n+51)/10

#### **3- MASS VALUES FOR CREW MEMBERS**

#### According to AMC2 CAT.POL.MAB.100(d) Mass and balance, loading

(a) The operator should use the following mass values for crew to determine the dry operating mass:

(1) actual masses including any crew baggage; or

(!) (2) standard masses, including hand baggage, of **85 kg for flight** crew/technical crew members and **75 kg for cabin crew members**.

(b) The operator should correct the dry operating mass to account for any additional baggage. The position of this additional baggage should be accounted for when establishing the centre of gravity of the aeroplane.

# 4- MASS VALUES FOR PASSENGERS AND BAGGAGE According to AMC1 CAT.POL.MAB.100(e) Mass and balance, loading

(a) When the number of passenger seats available is:

- (1) less than 10 for aeroplanes; or
- (2) less than 6 for helicopters,

passenger mass may be calculated on the basis of a statement by, or on behalf of, each passenger, adding to it a predetermined mass to account for hand baggage and clothing. (!) The predetermined mass for hand baggage and clothing should be established by the operator on the basis of studies relevant to his particular operation. In any case, it should not be less than:

# (1) 4 kg for clothing; and

# (2) 6 kg for hand baggage.

The passengers' stated mass and the mass of passengers' clothing and hand baggage should be checked prior to boarding and adjusted, if necessary. The operator should establish a procedure in the operations manual when to select actual or standard masses and the procedure to be followed when using verbal statements.

(b) When determining the actual mass by weighing, passengers' personal belongings and hand baggage should be included. Such weighing should be conducted immediately prior to boarding the aircraft.

(c) When determining the mass of passengers by using standard mass values, the standard mass values in Tables 1 and 2 below should be used. The standard masses include hand baggage and the mass of any infant carried by an adult on one passenger seat. Infants occupying separate passenger seats should be considered as children for the purpose of this AMC. When the total number of passenger seats available on an aircraft is 20 or more, the standard masses for males and females in Table 1 should be used. As an alternative, in cases where the total number of passenger seats available is 30 or more, the 'All Adult' mass values in Table 1 may be used.

# (!) According to ANNEX I DEFINITIONS FOR TERMS USED IN ANNEXES II TO VIII

(5) For the purpose of passenger classification:

(a) 'adult' means a person of an age of 12 years and above;

(b) 'child/children' means persons who are of an age of two years and above but who are less than 12 years of age;

(c) 'infant' means a person under the age of two years;

# (!) Table 1

Standard masses for passengers — aircraft with a total number of passenger seats of 20 or more

Deccencer coste:	20 and	30 and more	
Passenger seats:	Male	Female	All adult
All flights except holiday charters	88 kg	70 kg	84 kg
Holiday charters(*)	83 kg	69 kg	76 kg
Children	35 kg	35 kg	35 kg

(\*) Holiday charter means a charter flight that is part of a holiday travel package. On such flights the entire passenger capacity is hired by one or more charterer(s) for the carriage of passengers who are travelling, all or in part by air, on a round- or circle-trip basis for holiday purposes. The holiday charter mass values apply provided that not more than 5 % of passenger seats installed in the aircraft are used for the non-revenue carriage of certain categories of passengers. Categories of passengers such as company personnel, tour operators' staff, representatives of the press, authority officials, etc. can be included within the 5% without negating the use of holiday charter mass values.

# (!) Table 2

Standard masses for passengers — aircraft with a total number of passenger seats of 19 or less

Passenger seats:	1 - 5	6 - 9	10 - 19
Male	104 kg	96 kg	92 kg
Female	86 kg	78 kg	74 kg
Children	35 kg	35 kg	35 kg

(!) (1) On aeroplane flights with 19 passenger seats or less and all helicopter flights where no hand baggage is carried in the cabin or where hand baggage is accounted for separately, 6 kg may be deducted from male and female masses in Table 2. Articles such as an overcoat, an umbrella, a small handbag or purse, reading material or a small camera are not considered as hand baggage.

(2) For helicopter operations in which a survival suit is provided to passengers,3 kg should be added to the passenger mass value.

(d) Mass values for baggage

(1) Aeroplanes. When the total number of passenger seats available on the aeroplane is 20 or more, the standard mass values for checked baggage of Table 3 should be used.

(2) Helicopters. When the total number of passenger seats available on the helicopters is 20 or more, the standard mass value for checked baggage should be 13 kg.

(3) For aircraft with 19 passenger seats or less, the actual mass of checked baggage should be determined by weighing.

## (!) Table 3

Standard masses for baggage — aeroplanes with a total number of passenger seats of 20 or more

Type of flight	Baggage standard mass
Domestic	11 kg
Within the European region	13 kg
Intercontinental	15 kg
All other	13 kg

(4) For the purpose of Table 3:

(i) domestic flight means a flight with origin and destination within the borders of one State;

(ii) flights within the European region mean flights, other than domestic flights, whose origin and destination are within the area specified in (d)(5); and

(iii) intercontinental flight means flights beyond the European region with origin and destination in different continents.

(5) Flights within the European region are flights conducted within the following area:

- N7200 E04500
- N4000 E04500
- N3500 E03700
- N3000 E03700
- N3000 W00600
- N2700 W00900
- N2700 W03000
- N6700 W03000
- N7200 W01000
- N7200 E04500

as depicted in Figure 1.

**Figure 1** The European region



(f) Other standard masses may be used provided they are calculated on the basis of a detailed weighing survey plan and a reliable statistical analysis method is applied. The operator should advise the competent authority about the intent of the passenger weighing survey and explain the survey plan in general terms. The revised standard mass values should only be used in circumstances comparable with those under which the survey was conducted. Where the revised standard masses exceed those in Tables 1, 2 and 3 of, then such higher values should be used.

(g) On any flight identified as carrying a significant number of passengers whose masses, including hand baggage, are expected to significantly deviate from the standard passenger mass, the operator should determine the actual mass of such passengers by weighing or by adding an adequate mass increment.

(h) If standard mass values for checked baggage are used and a significant number of passengers checked baggage is expected to significantly deviate from the standard baggage mass, the operator should determine the actual mass of such baggage by weighing or by adding an adequate mass increment.

### 5- FUEL POLICY

# According to AMC1 CAT.OP.MPA.150(b) PLANNING CRITERIA — AEROPLANES

The operator should base the defined fuel policy, including calculation of the amount of fuel to be on board for departure, on the following planning criteria:

(a) Basic procedure

# (!) The usable fuel to be on board for departure should be the sum of the following:

(!) (1) Taxi fuel, which should not be less than the amount expected to be used prior to take-off.

Local conditions at the departure aerodrome and auxiliary power unit (APU) consumption should be taken into account.

(!) (2) Trip fuel, which should include:

(i) fuel for take-off and climb from aerodrome elevation to initial cruising level/altitude, taking into account the expected departure routing;

(ii) fuel from top of climb to top of descent, including any step climb/descent;

(iii) fuel from top of descent to the point where the approach is initiated, taking into account the expected arrival procedure; and

(iv) fuel for approach and landing at the destination aerodrome.

(!) (3) Contingency fuel, except as provided for in (b), which should be the higher of:

(!) (i) Either:

# (!) (A) 5 % of the planned trip fuel or, in the event of in-flight replanning, 5 % of the trip fuel for the remainder of the flight;

(B) not less than 3 % of the planned trip fuel or, in the event of in-flight replanning, 3 % of the trip fuel for the remainder of the flight, provided that an en-route alternate (ERA) aerodrome is available;

(C) an amount of fuel sufficient for 20 minutes flying time based upon the planned trip fuel consumption, provided that the operator has established a fuel consumption monitoring programme for individual aeroplanes and uses valid data determined by means of such a programme for fuel calculation; or

(D) an amount of fuel based on a statistical method that ensures an appropriate statistical coverage of the deviation from the planned to the actual trip fuel.

This method is used to monitor the fuel consumption on each city pair/aeroplane combination and the operator uses this data for a statistical analysis to calculate contingency fuel for that city pair/aeroplane combination;

# (!) (ii) or an amount to fly for 5 minutes at holding speed at 1 500 ft (450 m), above the destination aerodrome in standard conditions.

### (!) (4) Alternate fuel, which should:

(i) include:

(A) fuel for a missed approach from the applicable DA/H or MDA/H at the destination aerodrome to missed approach altitude, taking into account the complete missed approach procedure;

(B) fuel for climb from missed approach altitude to cruising level/altitude, taking into account the expected departure routing;

(C) fuel for cruise from top of climb to top of descent, taking into account the expected routing;

(D) fuel for descent from top of descent to the point where the approach is initiated, taking into account the expected arrival procedure; and

(E) fuel for executing an approach and landing at the destination alternate aerodrome;

(ii) where two destination alternate aerodromes are required, be sufficient to proceed to the alternate aerodrome that requires the greater amount of alternate fuel.

(!) (5) Final reserve fuel, which should be:

(!) (i) for aeroplanes with reciprocating engines, fuel to fly for 45 minutes; or

(!) (ii) for aeroplanes with turbine engines, fuel to fly for 30 minutes at holding speed at 1 500 ft (450 m) above aerodrome elevation in standard conditions, calculated with the estimated mass on arrival at the destination alternate aerodrome or the destination aerodrome, when no destination alternate aerodrome is required.

(!) (6) The minimum additional fuel, which should permit:

(i) the aeroplane to descend as necessary and proceed to an adequate alternate aerodrome in the event of engine failure or loss of pressurisation, whichever requires the greater amount of fuel based on the assumption that such a failure occurs at the most critical point along the route, and

(A) hold there for 15 minutes at 1 500 ft (450 m) above aerodrome elevation in standard conditions; and

(B) make an approach and landing, except that additional fuel is only required if the minimum amount of fuel calculated in accordance with (a)(2) to (a)(5) is not sufficient for such an event; and

(ii) holding for 15 minutes at 1 500 ft (450 m) above destination aerodrome elevation in standard conditions, when a flight is operated without a destination alternate aerodrome.

(!) (7) Extra fuel, which should be at the discretion of the commander.

## (!) Simplified Notes:

The usable fuel to be on board for departure should be the sum of the following:

- Taxi fuel, to taxi from the ramp to the runway to start the take-off
- Trip fuel, from take-off at departure to landing at destination
- **Contingency fuel**, except as provided for in, which should be the higher of:
  - > 5 % of the planned trip fuel or
  - an amount to fly for 5 minutes at holding speed at 1 500 ft (450m), above the destination aerodrome in standard conditions.
- Alternate fuel
- Final reserve fuel, which should be:
  - for aeroplanes with reciprocating engines, fuel to fly for 45 minutes; or
  - for aeroplanes with turbine engines, fuel to fly for 30 minutes at holding speed at 1 500 ft (450 m) above aerodrome elevation in standard conditions
- The minimum additional fuel,
  - Allow to achieve published requirement in the event of ta failure of the critical engine
  - To hold above destination aerodrome when operating without alternate aerodrome
- **Extra fuel**, which should be at the discretion of the commander

#### 6- Fuel Conversion

The fuel is measured as a volume when establishing the required of quantity, the common unit used in aviation for the fuel quantity are:

- US Gallon (USG)
- Imperial Gallon (ImpG)
- Litre (I)

### According to ICAO Annex 5

However, in order to establish the mass and balance of the aeroplane during flight planning, it is necessary to measure the mass of the fuel. The common unit used in aviation for the fuel mass are:

- Kilogram (kg)
- Pound (lb)

## According to ICAO Annex 5

#### 1 kg = 2,205 lbs

To convert a quantity (v) to a mass (m), the density ( $\rho$ ) of the substance must be known.

$$v x \rho = v x \frac{m}{v} = v x \frac{m}{v} = m$$

## Specific Gravity (SG)

SG is the ratio of the density of a substance to the density of a reference substance. Here it will be the ratio between the density of the fuel ( $\rho_{fuel}$ ) to the density of the water ( $\rho_{water}$ )

SG = 
$$\rho_{fuel} / \rho_{water}$$



**NOTE!** A common conversion of AvGas 100LL is: **1 USG of AvGas 100LL \leftrightarrow 6 lbs** 

# III) FUNDAMENTALS OF CG CALCULATION

### A- CG POSITIONNING

#### 1- Moment

The **moment** or **moment of force** is a rotational force. In other words, a moment, when it exists, will rotate a plan around its centre of pivot.

A moment will exist when a force is applied over a plan. The bigger the force the bigger the moment. The moment depends also on the distance from the centre of pivot where the force has been applied. This distance is known as the "arm". The longer the arm the bigger the moment. If the force is applied over the centre of pivot, there will be no rotation.



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Moment [N.in] = F [N]x arm [in]
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When an object with a given mass (m) is subject to its Weight (W=m.g). The weight is a force acting downward toward the centre of the Earth.

So when an object is placed over a plan, the Moment generated to that plan will depend on the arm from the object and the centre of pivot, as well on the mass (m) of that object.



Moment [kg.in] = m [kg]x arm [in]

Now let's consider the two sides of the centre of pivot for the same plan.



When a mass is placed on the right side of the centre of pivot, the moment will rotate the plan clockwise, when a mass is placed on the right side of the centre of pivot, the moment will rotate the plan counter-clockwise.

To be able to distinguish between those two moments, the centre of pivot will become the origin of the plan (arm = 0).

All masses located before the centre of pivot have a negative arm and so generate a negative moment.

All masses located after the centre of pivot have a positive arm and so generate a positive moment.



Let's place some masses on the plan:



In this case, the mass has been put on the same arm on both side, their moments are:

M<sub>A</sub> = 5 kg x (-6in) = -30 kg.in

 $M_B = 5 \text{ kg x (+6in)} = +30 \text{ kg.in}$ 

In this case, both moments are cancelling each other's, there will be no rotation. The total moment is:

#### $\mathbf{M}_{\text{TOT}} = \mathbf{M}_{\text{A}} + \mathbf{M}_{\text{B}} = \mathbf{0}$

Equivalently, it is like the both masses are located on the centre of pivot (arm=0). If both masses were located on the centre of pivot, the CG, where the total weight is applied, would be located on the centre of pivot ( $CG_{ARM}=0$ ). Consequently if the total mass would be applied on the centre of pivot, the final or total moment is nil.

M<sub>TOT</sub> = m<sub>TOT</sub> x CG<sub>ARM</sub> = (5 kg + 5 kg) x 0 in = 10 kg x 0 in = 0

The total moment, it's like ONE moment from ONE mass and the CG is the location of that ONE mass.

→ So the CG location, or CG arm, can be found by the following formula:

$$CG_{ARM} = M_{TOT} / m_{TOT}$$

#### The Centre of Gravity (CG) is:

- the point where the total weight of an object is said to act through
- the point of balance
- the point of rotation

When an object is held over its CG, the object is in equilibrium (no rotation)



#### 2- Spreading masses



Here the moments generated from each mass are:

M<sub>A</sub> = 20 kg x (-10in) = -200 kg.in

 $M_B = 10 \text{ kg x } (+2in) = +20 \text{ kg.in}$ 

The total moment is:

#### M<sub>TOT</sub> = M<sub>A</sub> + M<sub>B</sub> = (-200) + (+20) = -180 kg.in

The negative result means that the total rotation is counter-clockwise



This final moment would be the same as the total mass would have been applied through the CG. **Like ONE mass applied generating ONE moment**. To find the CG position:

$$M_{TOT} = M_A + M_B = (-200) + (+20) = -180$$
 kg.in

 $CG_{ARM} = M_{TOT}/m_{TOT}$ 

#### $m_{TOT} = m_A + m_B = 20 + 10 = 30 \text{ kg}$

CG<sub>ARM</sub> = (-180)/30 = -6 in

The negative result of the CG<sub>ARM</sub> means that the CG is located before the centre of pivot.



#### 3- Adding mass



To calculate the total moment, it is the sum of the moments generated from each mass:

M<sub>A</sub> = 20 kg x (-10in) = -200 kg.in

M<sub>B</sub> = 10 kg x (+2in) = +20 kg.in

M<sub>c</sub> = 50 kg x (+6in) = +300 kg.in

 $M_{TOT} = M_A + M_B + M_C = (-200) + (+20) + (+300) = +120 \text{ kg.in}$ 

The positive result means that the rotation will be clockwise



The positive result of the CG<sub>ARM</sub> means that the CG is located after the centre of pivot. **The CG shifted by +7,5in or 7,5in AFT** [+1,5-(-6) = +7,5]



However, remember before adding the mass "C", the masses "A" and "B" where acting like ONE mass (initial  $m_{TOT}$ ), located at the initial CG<sub>ARM</sub>, generating ONE moment (initial  $M_{TOT}$ ).

Adding the mass "C" would result in "two" masses on the plan generating two moments:

- The initial total mass (ONE mass) generating the total moment (ONE moment) through the initial CG<sub>ARM</sub> located at +1,5in, and
- The mass "C" generating a moment "C" through the location +6in



The new total moment from those "two" masses is:

Initial M<sub>TOT</sub> = 30 kg x (-6in) = -180 kg.in

M<sub>c</sub> = 50 kg x (+6in) = +300 kg.in

newM<sub>TOT</sub> = initM<sub>TOT</sub> + M<sub>C</sub> = (-180) + (+300) = +120 kg.in

The new CG position is:

newM<sub>TOT</sub> = initM<sub>TOT</sub> + M<sub>C</sub> = (-180) + (+300) = +120 kg.in

newCG<sub>ARM</sub> = newM<sub>TOT</sub>/new m<sub>TOT</sub> CG<sub>ARM</sub> = (+120)/80 = +1,5 in

new m<sub>τoτ</sub> = init m<sub>τoτ</sub> + m<sub>c</sub> = 30 + 50 = 80 kg

Same result as previous calculations

→ newCG<sub>ARM</sub> = [initM<sub>TOT</sub> + (m<sub>ADDED</sub> x arm<sub>m</sub>)] / (init m<sub>TOT</sub> + m<sub>ADDED</sub>)

This formula can be used, when rearranged to find out the mass to be added to shift the CG to a new desired arm (location) from the centre of pivot

(init m<sub>TOT</sub> + m <sub>TO ADD</sub>) x newCG<sub>ARM</sub> = (init m<sub>TOT</sub> x initCG<sub>ARM</sub>) + (m <sub>TO ADD</sub> x arm<sub>m</sub>)

#### 4- Removing mass



To calculate the total moment, it is the sum of the moments generated from each mass:

 $M_A = 8 \text{ kg x} (-10 \text{ in}) = -80 \text{ kg.in}$ 

 $M_B = 10 \text{ kg x } (+2\text{in}) = +20 \text{ kg.in}$ 

M<sub>c</sub> = 50 kg x (+6in) = +300 kg.in

#### $M_{TOT} = M_A + M_B + M_C = (-80) + (+20) + (+300) = +240$ kg.in

The positive result means that the rotation will be clockwise



The positive result of the CG<sub>ARM</sub> means that the CG is located after the centre of pivot. The CG shifted by +2,0in or 2,0in AFT [+3,5-(+1,5) = +2,0]



However, remember before removing from the mass "A", the masses "A", "B" and "C" where acting like ONE mass (initial  $m_{TOT}$ ), located at the initial CG<sub>ARM</sub>, generating ONE moment (initial  $M_{TOT}$ ).

Removing from the mass "A" would result in "two" masses on the plan generating two moments:

- The initial total mass (ONE mass) generating the total moment (ONE moment) through the initial CG<sub>ARM</sub> located at +1,5in, and
- A "removed" mass generating a "removed moment" at the location -10in



The new total moment from those "two" masses is:

- Initial M<sub>TOT</sub> = 80 kg x (+1,5in) = +120 kg.in
- ➤ M<sub>REMOVED</sub> = -12 kg x (-10in) = +120 kg.in

newM<sub>TOT</sub> = initM<sub>TOT</sub> + M<sub>REMOVED</sub> = (+120) + (+120) = +240 kg.in

The new CG position is:

newM<sub>TOT</sub> = initM<sub>TOT</sub> + M<sub>REMOVED</sub> = (+120) + (+120) = +240 kg.in

newCG<sub>ARM</sub> = newM<sub>TOT</sub>/new m<sub>TOT</sub>

new m<sub>TOT</sub> = init m<sub>TOT</sub> – m<sub>REMOVED</sub>

 $CG_{ARM} = (+240)/68 = +3,5$  in

= 80 - 12 = 68 kg

Same result as previous calculations

→ newCG<sub>ARM</sub> = [initM<sub>TOT</sub> - (m<sub>REMOVED</sub> x arm<sub>m</sub>)] / (init m<sub>TOT</sub> - m<sub>REMOVED</sub>)

This formula can be used, when rearranged to find out the mass to be removed to shift the CG to a new desired arm (location) from the centre of pivot

(init mTOT - m TO REMOVE) x newCGARM = (init mTOT x initCGARM) - (m TO REMOVE x armm)

#### 5- Repositioning mass



**Shift = arm\_{NEW} - arm\_{OLD} = (-10) - (+6) = -16in** 

(A negative result means a FWD SHIFT and positive result means AFT SHIFT)

To calculate the total moment, it is the sum of the moments generated from each mass:

 $M_{A} = 28 \text{ kg x (-10in)} = -280 \text{ kg.in}$   $M_{B} = 10 \text{ kg x (+2in)} = +20 \text{ kg.in}$   $M_{C} = 30 \text{ kg x (+6in)} = +180 \text{ kg.in}$  $M_{TOT} = M_{A} + M_{B} + M_{C} = (-280) + (+20) + (+180) = -80 \text{ kg.in}$ 

The negative result means that the rotation will be couter-clockwise



The positive result of the CG<sub>ARM</sub> means that the CG is located after the centre of pivot. **The CG shifted by +4,7in or 4,7in FWD** [-1,2 - (+3,5) = -4,7]



However, remember before repositioning the mass, the masses "A", "B" and "C" where acting like ONE mass (initial  $m_{TOT}$ ), located at the initial CG<sub>ARM</sub>, generating ONE moment (initial  $M_{TOT}$ ). Repositioning will not affect the ONE mass, the total mass remains the same. However during the repositioning, the mass will be removed from one location, generating a "*removed*" moment on that location, but when added to the new location, it will generate a moment on the new location. The difference between those two moments will be the moment generated by the repositioned mass.

Repositioning a mass would result in "two" final" moments:

- The initial total mass (ONE mass) generating the total moment (ONE moment) through the initial CG<sub>ARM</sub> located at +3,5in, and
- The difference between the generated moment at the location +6in and the *"removed"* moment at the location -10in by the mass repositioned.



The new total moment from those "two final" moments is:

- Initial M<sub>TOT</sub> = 68 kg x (+3,5in) = +240 kg.in
- ➢ MFROM THE SHIFT = MREMOVED FROM OLD LOCATION − MGENERATED AT NEW LOCATION
  - = (m<sub>REPOSIONNED</sub> x arm<sub>NEW</sub>) (m<sub>REPOSITIONNED</sub> x arm<sub>OLD</sub>)
  - = m<sub>REPOSIONNED</sub> x (arm<sub>NEW</sub> arm<sub>OLD</sub>) = m<sub>REPOSIONNED</sub> x shift
  - = 20 kg x (-16in) = -320 kg.in

newM<sub>TOT</sub> = initM<sub>TOT</sub> + M<sub>FROM THE SHIFT</sub> = (+240) + (-320) = -80 kg.in

The new CG position is:

<b>newM</b> TOT = <b>initM</b> TOT + <b>M</b> FROM THE SHIFT	newCG <sub>ARM</sub> = newM <sub>TOT</sub> /new m <sub>TOT</sub>
= (+240) + (-320) = -80 kg.in	CG <sub>ARM</sub> = (-80)/68 = -1,2 in
<b>m</b> тот <b>= 68 kg</b>	Same result as previous calculation

→ newCG<sub>ARM</sub> = [initM<sub>TOT</sub> + (m<sub>REPOSITIONED</sub> x shift)] / m<sub>TOT</sub>

This formula can be used, when rearranged to find out the mass to be repositioned to shift the CG to a new desired arm (location) from the centre of pivot.

- → newCG<sub>ARM</sub> = [(m<sub>TOT</sub> x initCG<sub>ARM</sub>) + (m<sub>REPOSITIONED</sub> x shift)] / m<sub>TOT</sub>
- → newCG<sub>ARM</sub> = [(m<sub>TOT</sub> x initCG<sub>ARM</sub>)/ m<sub>TOT</sub>]+ [(m<sub>REPOSITIONED</sub> x shift) / m<sub>TOT</sub>]
- → newCG<sub>ARM</sub> = initCG<sub>ARM</sub> + [(m<sub>REPOSITIONED</sub> x shift) / m<sub>TOT</sub>]
- → newCG<sub>ARM</sub> initCG<sub>ARM</sub> = [(m<sub>REPOSITIONED</sub> x shift) / m<sub>TOT</sub>]
- → CG<sub>SHIFT</sub> = (m<sub>REPOSITIONED</sub> x shift) / m<sub>TOT</sub>
- → CG<sub>SHIFT</sub> x m<sub>TOT</sub> = m<sub>REPOSITIONED</sub> x shift

```
CGNEED TO SHIFT X MTOT = M TO REPOSITION X shift
```

#### Summary of this sub-part

A moment is rotation force, it is the result of a mass located at a certain arm from the centre of pivot on a plan.

#### **MOMENT** = mass x arm

When the mass is located before the centre of pivot, it will generate a negative moment, and when the mass is located after the centre of pivot, it will generate a positive moment.

When several masses are located on the same plan, each mass will generate a moment. The final moment will result from the sum of the moment generated. This final moment would be the same if the total mass would have been applied through the centre of gravity (CG), which is the centre of equilibrium. The location of the CG could be found with the MOMENT FORMULA

#### mass<sub>TOTAL</sub> x CG<sub>arm</sub> = Moment<sub>TOTAL</sub>

#### **CG**arm = **Moment**TOTAL / **mass**TOTAL

When adding, removing or repositioning a mass, the CG will shift along the plan. The following formulas summarise the demonstrations, they don't need to me learnt for the moment, they will be review and simplified later in the lesson.

Adding mass: (init m<sub>TOT</sub> + m <sub>TO ADD</sub>) x newCG<sub>ARM</sub> = (init m<sub>TOT</sub> x initCG<sub>ARM</sub>) + (m <sub>TO ADD</sub> x arm<sub>m</sub>)

#### Removing mass:

(init mTOT - m TO REMOVE) x newCGARM = (init mTOT x initCGARM) - (m TO REMOVE x armm)

#### Shifting mass:

**CG**ARM NEED TO SHIFT **X M**TOT **= M** TO REPOSITION **X shift**MASS

#### **B- CG LOCATION & REFERENCE**

In the previous calculation, all arms measured before the centre of pivot were negative, and all arms measured after the centre of pivot were positive. So the centre of pivot was the reference of measurement, known as **datum (arm=0)**.



The datum has been chosen there to simplify the calculation and to understand the fundamental of the CG location. However the datum doesn't have to be necessary on the centre of pivot. The scale can be shifted, causing all the arms to change, **but the calculation to define the CG position would still be the same** because the arms would all have been modified by the same scale. i.e:



When an arm is measured from the datum, it's called Balance Arm (BA).

Let's use come back on the example where the total mass was 30 kg. Remember, for this configuration, the CG were at (-6,0in) or 0,6in before the datum and the centre of pivot.



Now let's define the CG position by using a new datum position.



 $M_{A} = m_{A} \times BA_{A} = 20 \text{ kg x (-6in)} = -120 \text{ kg.in}$   $M_{B} = m_{B} \times BA_{B} = 10 \text{ kg x (+6in)} = +60 \text{ kg.in}$   $M_{TOT} = M_{A} + M_{B} = (-120) + (+60) = -60 \text{ kg.in}$   $m_{TOT} = m_{A} + m_{B} = 20 \text{ kg} + 10 \text{ kg} = 30 \text{ kg}$   $CG_{ARM} = M_{TOT}/m_{TOT} = -60 / 30 = -2,0 \text{ in}$ 

So according to this scale, the CG is located at -2,0in or **2,0in before the datum**.



As it can be seen, the CG, reference to the datum it doesn't have the same position, however it has the same position compared to centre of pivot (-6in) and so it is located on the same position on the plan.

So again, to determine the CG position of a plan, the datum doesn't have to be on the centre of pivot. I can be anywhere on the axis of the plan, as long as all the masses are measure from the same datum.

So, for an aircraft, the manufacturer will define its datum to measure all the masses (equipment, passengers, etc). The datum is be published in the AFM/POH of the aircraft.

#### **DEFINITION TO REMEMBER IN THIS SUB-PART**

#### Datum:

(relative to an aeroplane) is that point on the longitudinal axis (or extension thereof) from which the centres of gravities of all masses are referenced.

In other words, the datum is the plan perpendicular to the aircraft longitudinal axis (or extension of it).

The datum is determined and set by the manufacturer.

## Balance Arm (BA):

is the distance from the datum to the centre of gravity of a mass.

## Moment:

is the product of the mass and the balance arm.



### **C- NEUTRAL POINT**

It has been seen when the CG is located before the centre of pivot, the axis will rotate counter-clockwise.



Also, when the CG is located after the centre of pivot, the axis will rotate clockwise.



This rotation is mainly caused because of the total mass applied at a point (CG) different from the centre of pivot. However, if the CG were located at the centre of pivot, there will be no rotation of the axis.

Here the situation is neutral, and for that, the centre of pivot is known as the NEUTRAL POINT. When the CG is at the NEURAL POINT, there will be no rotation of the axis.

Although, if one mass, among many, is located on the neutral point, it doesn't mean that there will be no rotation, the rotation will only depend on the position of the CG compared to the neutral point, and on one mass among many.

When the CG is at the neutral point, if the datum were at the neutral point, all the moments of each force would cancel each other's. (Moment<sub>TOT</sub>=0)

 $\mathbf{F}_{\mathbf{A}} \mathbf{x} \mathbf{A} = \mathbf{F}_{\mathbf{B}} \mathbf{x} \mathbf{B}$ 

e.g.



#### **D- REPOSITION OF CG**

It has been seen in the INTRODUCTION, the CG must lie within the most FWD CG limit and most AFT CG limit. Otherwise the aircraft performances will be dramatically affected and the pilot will experience undesirable and dangerous flight characteristics.

It has been seen that the CG will shift by adding, removing or repositioning a mass. Two types of mathematical problems you can be led to resolve in this lesson:

- Find the mass to add, remove or reposition when the final and initial CG position are known.
- Find the final CG position with the initial CG position when the added, removed or repositioned mass known.

The equations seen before during the previous demonstrations can be used:

#### Adding mass:

(init m<sub>TOT</sub> + m <sub>TO ADD</sub>) x newCG<sub>ARM</sub> = (init m<sub>TOT</sub> x initCG<sub>ARM</sub>) + (m <sub>TO ADD</sub> x arm<sub>m</sub>)

#### **Removing mass:**

```
(init mTOT - m TO REMOVE) X newCGARM = (init mTOT X initCGARM) - (m TO REMOVE X armm)
```

#### Shifting mass:

**CG**ARM NEED TO SHIFT **X M**TOT **= M** TO REPOSITION **X shift**MASS

However, in this lesson the variables have been replaced by simplified symbols to make the equations easier to read:

m: Mass to be added, removed, or repositioned

M: Original mass of the aircraft

X: Original CG distance from the datum

**Y:** Final CG distance from the datum after the mass (m) has been added, removed or repositioned

Z: Distance from the datum that the mass (m) is to be added or removed

d: Distance the mass (m) has to move from a cargo to another

CC: Distance the CG needs to move (Y-X)

Adding mass:

 $(M+m) \times Y = (M \times X) + (m \times Z)$ 

**Removing mass:** 

 $(M-m) \times Y = (M \times X) - (m \times Z)$ 

Shifting mass:

 $m \times d = M \times CC$ 

#### Examples

### Question 1: Find new CG position after adding a mass

An aircraft has a total mass of 5430 kg with a CG of 5,43m aft of the datum line. Minutes before despatch an additional load of 210kg was loaded onto the aircraft's cargo hold located at 8,92m aft of the datum line. What's the aircraft new CG position?

#### Solution:

 $(M+m) \times Y = (M \times X) + (m \times Z) \leftrightarrow$ 

# Y = [(M x X) + (m x Z)] / (M+m)

Y = [(5430 kg x 5,43m) + (210 kg x 8,92m)] / (5430 kg + 210 kg)

Y = (29 484,9 kg.m + 1 873,2 kg.m) / 5640 kg = 31 358,1 kg.m / 5640 kg

Y ≈ +5,56m

 $\rightarrow$  After the additional load, the aircraft new CG position is 5,56m AFT the datum

#### Question 2: Find the mass to add to obtain the new CG position

An aircraft has a total mass of 5430 kg with a CG of 5,12m aft of the datum line. However its FWD CG LIMIT is 5,20m aft of the datum line. How much mass must be added onto the aircraft's cargo hold located at 8,92m aft the datum line to shift the CG to its most aft allowed position?

#### Solution:

 $(\mathsf{M}\text{+}\mathsf{m}) \ge \mathsf{Y} = (\mathsf{M} \ge \mathsf{X}) + (\mathsf{m} \ge \mathsf{Z}) \longleftrightarrow \mathsf{Y}.\mathsf{M} + \mathsf{Y}.\mathsf{m} = \mathsf{M}.\mathsf{X} + \mathsf{m}.\mathsf{Z} \longleftrightarrow$ 

 $Y.m - m.Z = M.X - Y.M \leftrightarrow m.(Y-Z) = M.(X-Y) \leftrightarrow$ 

#### m = [M.(X-Y)] / (Y-Z)

m = [5430 kg x (5,12m - 5,20m)] / (5,20m - 8,92m)

m = [5430 kg x (-0,08m)] / (-3,72m) = (-434,4 kg.m) / (-3,72m)

#### m = 117 kg

→ To shift the CG to its most AFT limit, a mass of 117kg must be added onto the cargo hold located at 8,92m aft the datum line.

# Question 3: Find new CG position after removing a mass

An aircraft has a total mass of 5430 kg with a CG of 5,43m aft of the datum line. If the trip fuel is of 210kg with the fuel tanks located at 8,92m aft of the datum line. What's the aircraft CG position upon landing?

# Solution:

 $(M-m) \ge Y = (M \ge X) - (m \ge Z) \iff$ 

 $Y = [(M \times X) - (m \times Z)] / (M-m)$ 

Y = [(5430 kg x 5,43m) - (210 kg x 8,92m)] / (5430 kg - 210 kg)

Y = (29 484,9 kg.m - 1 873,2 kg.m) / 5220 kg = 27 611,7 kg.m / 5220 kg

# Y ≈ +5,29m

→ After landing, the aircraft new CG position is 5,30m AFT the datum

# Question 4: Find the mass to remove to obtain the new CG position

An aircraft has a total mass of 6100 kg with a CG of 5,62m aft of the datum line. However its AFT CG LIMIT is 5,50m aft of the datum line. How much mass must be removed onto the aircraft's cargo hold located at 8,50m aft the datum line to shift the CG to its most aft allowed position?

## Solution:

 $(\mathsf{M}\text{-}\mathsf{m}) \ge \mathsf{Y} = (\mathsf{M} \ge \mathsf{X}) - (\mathsf{m} \ge \mathsf{Z}) \longleftrightarrow \mathsf{Y}.\mathsf{M} - \mathsf{Y}.\mathsf{m} = \mathsf{M}.\mathsf{X} - \mathsf{m}.\mathsf{Z} \longleftrightarrow$ 

 $\mathsf{m.Z} \operatorname{-} \mathsf{Y.m} = \mathsf{M.X} \operatorname{-} \mathsf{Y.M} \longleftrightarrow \mathsf{m.(Z-Y)} = \mathsf{M.(X-Y)} \longleftrightarrow$ 

## m = [M.(X-Y)] / (Z-Y)

m = [6100 kg x (5,62m - 5,50m)] / (8,50m - 5,50m)

m = [6100 kg x (0,12m)] / (3,42m) = (732 kg.m) / 3,42m

## m = 244 kg

→ To shift the CG to its most AFT limit, a mass of 244kg must be removed from the cargo hold located at 8,50m aft the datum line.

### Question 5: Find new CG position after repositioning a mass

An aircraft has a total mass of 180 000 kg with a CG of 15,56m aft of the datum line. Minutes before despatch a load of 1000kg was transferred from the aircraft's cargo hold 4 located at 21,13m aft of the datum line to the cargo hold 1 located at 2,73m aft of the datum line. What's the aircraft new CG position?

#### Solution:

 $\mathsf{m} \mathsf{x} \mathsf{d} = \mathsf{M} \mathsf{x} \mathsf{C} \leftrightarrow$ 

C = m x d / M

1) Find «d», the distance the mass (m) has been moved from a cargo to another

d = Final mass position – Original mass position

d = (+2,73) – (+21,13) = -18,4m  $\rightarrow$  the mass has been moved 18,4m forward

2) Find «C», the CG shift

C = m x d / M

C = 1000 kg x (-18,4) / 180 000 kg = (-18 400 kg.m) / 180 000

C ≈ (-0,10m)

→ After transferring the mass from cargo hold 4 to cargo hold 1, the CG shifted by (-0,10m) or 0,10m forward.

New CG position = +15, 56m + (-0,10m) = +15,46m



## Question 6: Find the mass to reposition to obtain the new CG position

An aircraft has a total mass of 180 000 kg with a CG of 15,75m aft of the datum line. The AFT CG limit is 15,65m aft of the datum line. What mass must be transferred between the aircraft's cargo hold 4 located at 21,13m aft of the datum line and the cargo hold 1 located at 2,73m aft of the datum line, to shift the CG to its AFT limit position?

#### Solution:

 $m x d = M x CC \leftrightarrow$ 

#### $m = M \times CC / d$

1) Find «d», the distance the mass (m) has been moved from a cargo to another

Obviously, since the CG has to move forward, the mass must be transferred forward, so from cargo hold 4 to cargo hold 1.

d = Final mass position – Original mass position

d =  $(+2,73) - (+21,13) = -18,4m \rightarrow$  the mass will be transferred by 18,4m forward

#### 2) Find «CC», the distance CG has to move

C = Final CG position – Original CG position

C = (+15,65m) - (+15,75m) = (-0,10m)

3) Find the (m) to transfer

#### $m = M \times C / d$

m = 180 000 kg x (-0,10m) / (-18,4m) = (-18 000 kg.m) / (18,4m)

#### m ≈ 978 kg

→ A mass of approximately 978kg must be transferred from cargo hold 4 to cargo hold 1 in order to shift the CG to its most AFT limit.



# E- MEAN AERODYNAMIC CHORD (%MAC)

It has been seen that the CG location is expressed as a distance from the datum. A negative distance means that the CG is located before the datum and a positive distance means that the CG is located after the datum.

In some aircraft, generally the Medium or Long Range Jet, their CG locations are expressed as percentage of their Mean Aerodynamic Chord (MAC) (Length of their average chordline).





**B:** Distance from the Datum to the leading edge (negative if the datum is after the leading edge)

C: Length of the MAC, distance between the leading edge and the trailing edge

$$CG \ as \ a \ \% MAC = \frac{A-B}{C} x \ 100$$

#### Example:

#### Question 1: Find the new CG position as a %MAC with the new mass

The planned take-off mass of an aeroplane is 190 000 kg, with its centre of gravity located at 29% MAC (Mean Aerodynamic Chord). Distance from reference point to leading edge = 14 m. Length of MAC = 4.6m. Shortly prior to engine start, the local staff informs the flight crew that an additional load of 4000kg must be loaded in cargo 4 (located at 24.26 m aft of the reference point). After loading this cargo, the new centre of gravity location will be (in %MAC):

#### Solution:

For adding mass, the correct equation to use is:

#### $(M+m) \times Y = (M \times X) + (m \times Z)$

#### 1) Find "X", the original CG<sub>ARM</sub> from the datum (reference point)

 $CG_{\%MAC} = (A-B) / C \times 100$ 

 $A = (CG_{\%MAC} / 100 \times C) + B$ 

A = (29/100 x 4,6m) + 14m = (0,29 x 4,6m) + 14m = 1,334m + 14m

A = +15,334 m  $\rightarrow$  the original CG is located at 15,334m after the datum

2) Now with the previous equation, determine "Y", the new CG<sub>ARM</sub>

 $(M+m) \times Y = (M \times X) + (m \times Z)$ 

 $Y = [(M \times X) + (m \times Z)] / (M+m)$ 

Y = [(190 000kg x [+15,334m]) + (4 000kg x [+24,26m])] / (190 000kg + 4 000kg)

Y = (2 913 460kg.m + 97 040kg.m) / 194 000kg

 $Y \approx$  +15,52m → the new CG is located at 15,52m after the datum

#### 3) And now determine the new $CG_{\rm \% MAC}$ with the new $CG_{\rm ARM}$

$$CG_{\%MAC} = (A-B) / C \times 100$$

 $CG_{\text{MAC}} = (+15,52m - 14m)/4,6m \times 100 = 1,52m/4,6m \times 100 \approx 0,33 \times 100$ 

#### CG<sub>%MAC</sub> ≈33%

ightarrow The new CG position is 33%MAC

# Question 2: Find the new mass with the CG position as a %MAC

An aircraft with a total mass of 200 000 kg, the centre of gravity (CG) is located at 32%MAC. The distance from the datum to the leading edge is 14m aft and the length of MAC is 4,6m.

For performance reasons, the captain decides to redistribute part of the cargo loading between cargo hold 1 located at 2,73m aft from the datum and cargo hold 4 located at 21,13m aft from the datum, in order to take off with a new CG location of 34%MAC. How much mass the captain must ask to transfer?

#### Solution:

For repositioning mass, the correct equation to use is:

#### m x d = M x C

#### 1) Find the CG to shift in %MAC

%MAC = New CG<sub>%MAC</sub> - Old CG<sub>%MAC</sub>

%MAC = 34% - 32%

%MAC = +2%  $\rightarrow$  The CG<sub>%MAC</sub> has to move 2% further aft of the MAC

#### 2) To find "CC", convert the CG shift from %MAC to ARM

CG ARM TO SHIFT = CG $_{MAC}$  TO SHIFT/100 x MAC

CG ARM TO SHIFT = +2/100 x 4,6m

CG ARM TO SHIFT = +0,092m  $\rightarrow$  The CG has to shift by 0,092m further aft.

#### 3) Find distance the mass (m) has to move – "d"

Obviously, since the CG has to move afterward, the mass must be transferred afterward, so from cargo hold 1 to cargo hold 4.

d = Final mass position – Original mass position

d = (+21,13) - (+2,73) = +18,4m  $\rightarrow$  the mass will be transferred by 18,4m aft

## 4) Now find "m", the mass to reposition with the previous equation

```
m x d = M x C
```

 $m = M \times C / d$ 

m = 200 000 kg x (+0,092m) / (+18,4m) = 18 400 kg.m / (+18,4)

m = 1000 kg

→ 1000 kg must be transferred from cargo hold 1 to cargo hold 4 in order to shift the CG to 34%MAC.

#### F- CG SHIFT IN FLIGHT

The pilots must be aware that the CG moves in flight. Those movements are mainly due to:

## 1- Fuel consumption

When the engines are consuming the fuel, the fuel quantity is reduced, and so the moment generated from the fuel is reduced. Thus cause the CG to shift.

For aeroplanes equipped with several fuel tanks (centres, inner wings and outer wings), fuel in the centre tank is fed to the engines initially and when the centre tanks is emptied, the fuel from the wing inner tank is taken. Fuel from the wing inner tank is used until fuel level in the inner tank drops down to certain value (i.e. 750 kgs for Airbus A320). When this limit is reached, two transfer valves open and let the fuel in outer tanks to be filed into the inner tanks. So wing outer tank fuel is used at last. There is a special reason to implement this sequence. The wing is subjected to upward bending force throughout the journey due to the huge lift force created. Heavy upward forces try to bend the wing upwards and weight of the fuel provides the counter-attack for this bending moment. As arm increase when moving towards the tip, even a less amount of fuel will create more counter force to keep the wings straight.



1) CENTRE TANK - 2) WING INNER TANKS - 3) WING OUTER TANKS

## 2- Landing gear extension/retraction

When the landing gears are extended or retracted, depending on the type, the nose gears will either rotate forward or afterward. When they rotate forward, they will generated a bigger forward moment causing the CG to shift FWD, and when they rotate afterward, they will generated a bigger afterward moment causing the CG to shift AFT.



American Airlines Boeing B737 – Nose Landing gears rotating forward during retraction



Aeroflot Tupolev TU154 – Nose Landing gears rotating afterward during retraction

#### 3- Cabin crewmembers performing their normal duties

During their normal duties, the cabin crew will displace the beverage carts (trolleys) which are significantly heavy and cause the CG to shift in the same direction.

NOTE: When one person moves on-board, its mass is negligible, so the CG will not move significantly, unless when many persons moves at the same time.

#### The 2010 Bandundu Filair Let L-410 crash

Occurred on August 25, 2010, after a Filair aircraft Let L-410 crashed on approach to Bandundu Airport in the Democratic Republic of the Congo, killing 20 people, leaving one survivor.

The aircraft was operating a passenger flight from N'Dolo Airport, Kinshasa to Bandundu Airport, Bandundu, Mai-Ndombe District. At 13:00 local time (12:00UTC), the aircraft crashed into a house approximately 1 kilometre short of the runway at Bandundu.

The lone survivor of the crash claimed that a crocodile hidden in a duffel bag had escaped. The frightened passengers then moved towards the front of the aircraft away from the crocodile. This affected the aircraft weight and balance leading to a loss of control.

#### 4- Flaps extension/retraction

Operating the flaps will also shift the CG. The direction of shift depends on the type of flaps.

The flower or slotted flaps, on the large aeroplanes, those surfaces are large and heavy, during extension they move afterward and shift the CG AFT, and during retraction they move forward and so shift the CG FWD.

Although the plain or split flaps rotates down during extension, their mass shift forward, however the displacement is very negligible and will not significantly affect the CG position.

# IV) AIRCRAFT HANDLING & MAINTENANCE

### **A- AIRPLANE WEIGHTING PROCEDURE**

The airplane weighting procedure consist to determine the Basic Empty Mass (BEM) of the aircraft and the CG position at the BEM.

#### 1) Preparation

During this procedure, the aircraft is parked in an enclosed hangar, cleaned, defueled, <u>the oil is not required to be drained</u>, the wheels are deflated, and the aircraft seats and equipment are set in a certain position (refer to the POH/AFM of the aircraft for more details).

#### 2) Levelling

The aircraft is levelled from its levelling points (determined by the manufacturer) and a scale under each gear is placed. (Nose or tail, Left and Right gears)

## 3) Weighting

The mass from each scale is recorded and the Basic Empty Mass is obtained from the total mass.

#### 4) Measuring

A horizontal measurement is made from the datum to each gear, or between the nose or tail gear to the main gears if the datum is not provided. From those measurements and the previous masses obtained, the aircraft's CG at its BEM can be determined.

#### Example

#### When the datum is provided

An aircraft has a datum 25 cm aft of the nose wheel. The weight on the nose wheel is 125 lbs. The main gear is 180 cm aft of the datum. The weight on each main gear is 3000 lbs. The distance between the nose gear and main gear is 205cm. Calculate the BEM and determine the CG position.



```
BEM = Nose wheel mass + Main gears mass
```

BEM = 125 lbs + (2 x 3 000 lbs)

BEM = 6 125 lbs

Moment<sub>TOTAL</sub> = Moment of the nose wheel + Moment of the main gears

**Moment**<sub>TOTAL</sub> = [125 lbs x (-25cm)] + [2x3 000 lbs x (+180)]

**Moment**<sub>TOTAL</sub> = (-3 125 lbs.cm) + (+1 080 000 lbs.cm)

Moment<sub>TOTAL</sub> = +1 076 875 lbs.cm

CG<sub>ARM</sub> = Moment<sub>TOTAL</sub> / BEM

CG<sub>ARM</sub> = 1 076 875 lbs.cm / 6 125 lbs

CG<sub>ARM</sub> ≈ +175,82 cm

→ The BEM is 6125 lbs and the CG is located at 175,82 cm aft the datum

## When the datum is not provided (same aircraft)

When the datum is not provided, the location of the CG is compared to the locations of the wheels. In this case, we can for example use the nose wheel as a reference point or datum.

## Moment<sub>TOTAL</sub> = Moment of the nose wheel + Moment of the main gears

**Moment**<sub>TOTAL</sub> = [125 lbs x 0)] + [2x3 000 lbs x (+205)]

**Moment**<sub>TOTAL</sub> = 0 + (+1 230 000 lbs.cm)

Moment<sub>TOTAL</sub> = +1 230 000 lbs.cm

CG<sub>ARM</sub> = Moment<sub>TOTAL</sub> / BEM

CG<sub>ARM</sub> = +1 230 000 lbs.cm / 6 125 lbs

#### CG<sub>ARM</sub> ≈ +200,82 cm

→ The BEM is 6125 lbs and the CG is located at 200,82 cm aft of the nose wheel

If we compare the position of the CG at the BEM with the previous example where the datum was provided. The CG was located at 175,82 cm aft the datum, and the datum was 25cm after the nose wheel. If we add those two values. We'll find out that CG is at the same location compared to the nose wheel. So it works!

# **B- CARGO HANDLING**

The floor of the cargo hold has a Maximum Total Load that should never be exceeded when loading, otherwise the floor might collapse under the excessive load.

However, the cargo floor is subject to other limitations.

# 1- Floor Running Load (or linear load)

The maximum Running load or linear load it's the maximum mass that can held for a given distance.

To know the running load of an object, it is calculated as the mass of the object divided by the distance it covers along the aircraft's floor's length.

e.g.: A cargo weighting 60 kg is 30cm length, 20cm width and 15cm height



This cargo can have 3 Running Load:

**Maximum Running Load** = cargo mass/shortest side = 60kg/15cm = 4kg/cm

Middle Running Load = cargo mass/middle side = 60kg/20cm = 3kg/cm

Minimum Running Load = cargo mass/longest side = 60kg/30cm = 2kg/cm



An object has three orientation on which it can be loaded, to ensure the minimum Running Load, it's better to load the object on its longest sides.

NOTE: A cubical object, will have the same Running Load on each side.

# 2- Distribution Floor Intensity (or area or floor load)

The maximum area or floor load it's the maximum mass that can held for a given area.

To know the floor intensity of an object, it is calculated as the mass of the object divided by the area it occupies in the aircraft's floor's length.

e.g.: A cargo weighting 60 kg is 30cm length, 20cm width and 15cm height



This cargo can have 3 Floor intensities:





15 cm

Middle Floor Intensity = cargo mass/medium area = 60kg/(15cm x 30cm) = 60kg/450cm<sup>2</sup>

**Middle Floor Intensity** ≈ 0,13 kg/cm<sup>2</sup>



Minimum Floor Intensity = cargo mass/largest area

= 60kg/(20cm x 30cm) = 60kg/600cm<sup>2</sup>

**Minimum Floor Intensity** ≈ 0,1 kg/cm<sup>2</sup>

An object has three orientation on which it can be loaded, to ensure the minimum Floor Intensity, it's better to load the object on its biggest area (longest sides)

NOTE: A cubical object, will have the same Floor Intensity on each side.

To simply and resolve the problems due to the orientations of the cargo, some cargos are loaded over pallets or containers that fit the cargo hold of the aircraft and ensure the Floor Intensity not exceeding the limitations of the cargo hold.

## (The containers and the pallets are known are Unit Load Devices, ULD)

E.g. A cubical cargo weighting 600 kg of 0,50m side must be loaded in an aeroplane in which the maximum floor loading for a cargo compartment is 750kg/m<sup>2</sup>



On any face of the object, the Floor Intensity is:

```
Floor Intensity = Cargo mass/area = 600 kg/0,50m<sup>2</sup>
```

Floor Intensity = 2400 kg/m<sup>2</sup>

So if this object is loaded into the cargo compartment, the maximum floor loading would be exceeded.

The minimum required surface on which the mass of the object must be spread on is:

Maximum Floor Intensity = Cargo mass/smallest area  $\leftrightarrow$ 

Smallest area = Cargo mass/Maximum Floor Intensity = 600 kg/750 kg.m<sup>-2</sup>

## Smallest area = 0,8m<sup>2</sup>



To make this cargo not exceeding the maximum floor loading of the cargo compartment, it must be loaded over a pallet of minimum 0,8m<sup>2</sup> surface.

i.e. (100cm\*80cm) or (160cm\*50cm) etc.

#### **3-** Conversion

#### Mass

1kg = 9,81N under normal gravitational conditions (1g)

1 kg = 2,205 lbs

#### Distance

1 ft = 12 in

1 m = 3,28 ft

m	dm cm		mm
1	0	0	0
0	1	0	0
0	0	1	0
0	0	0	1

#### Area

1 m<sup>2</sup> = 10,76 ft<sup>2</sup>

1 ft<sup>2</sup> = 144 in<sup>2</sup>

n	า <sup>2</sup>	dr	dm²		cm²		mm²	
	1	0	0	0	0	0	0	
	0	0	1	0	0	0	0	
	0	0	0	0	1	0	0	
	0	0	0	0	0	0	1	

#### Volume

1 dm<sup>3</sup> = 1 l

m³		dm <sup>3</sup>		cm <sup>3</sup>		r	nm	3		
	1	0	0	0	0	0	0	0	0	0
	0	0	0	1	0	0	0	0	0	0
	0	0	0	0	0	0	1	0	0	0
	0	0	0	0	0	0	0	0	0	1

		Ι	dl	cl	ml	
m <sup>3</sup>	dm <sup>3</sup>		cm <sup>3</sup>		3	mm <sup>3</sup>
		1	0	0	0	
			1	0	0	
				1	0	
					1	

## V) MASS AND BALANCE CALCULATION & LOAD SHEETS

#### A- SINGLE ENGINE PISTON

Refer to CAP696 - Section 2: Data for Single-Engine Piston Aeroplane (SEP1)

#### **B- MULTI-ENGINES PISTON**

*Refer to CAP696 - Section 3: Data for Light Twin-Engine Piston Aeroplane (MEP1)* 

C- MEDIUM-RANGE & LONG-RANGE JET TRANSPORT

Refer to CAP696 - Section 4: Data for Medium-Range Jet Transport (MRJT1)