III

MAP PROJECTION

Orthomorphism / Conformality

Of all the ideal properties, the only essential one is that navigation bearings must be “correct” and the critical property is that angles on the Earth must be represented correctly on the chart.

This property is critical to aviation - or indeed for navigation generally. If you draw a line joining 2 points on the chart and measure the angle but then find that this does not correspond to the true direction on the Earth, the chart is useless for navigation. You might previously have thought that if you measure a track off any map, it will correspond to Earth direction but this is not true for most charts. Those charts that do have this property are in the minority and are known as **orthomorphic or conformal charts**.

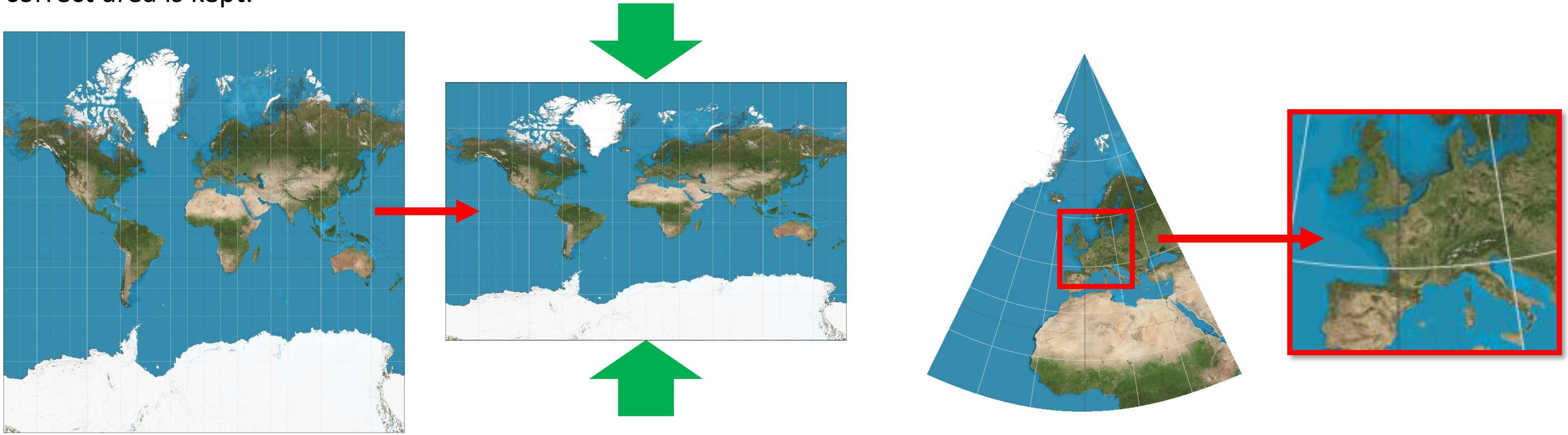
Remember that people produce charts for many reasons - not just navigation. For example, if your field of expertise is farming, then you might be interested in whether 1000 hectares of land in Kansas produces a greater or a lesser tonnage of wheat than 1000 hectares of land in Azerbaijan. In that case, the property you would look for in a map would be an accurate representation of area - but it would not matter to you whether bearings were represented accurately.

For a map to be orthomorphic or conform:
- Angles on the Earth’s surface should be represented by the same angles on the chart
- Scale should be constant
- Scale should be correct

Orthomorphism / Conformality

We briefly saw how maps or chart are produced, we saw that depending on the type of projection, the areas and the scale changes when away from the latitude of tangency (tangent to the globe during the projection).

Therefore, from the vast range of projections available, we have to select only those which are orthomorphic (or conformal). After the projection, to comply the maximum with orthomorphism, the map is either arranged mathematically or only the correct area is kept.



There are two fundamental conditions which must be met to achieve or satisfy orthomorphism/conformality.

- Meridians and parallels cut at right angles on the chart
- Scale at any point on the chart must be equal in all directions (or must change at an equal rate in all direction)

We can verify on the VFR Map of South Germany (ED-4) that the scale is correct and the same everywhere:

If we take 30' change of longitudes at 47°30'N, this distance is 20 Nm on Earth

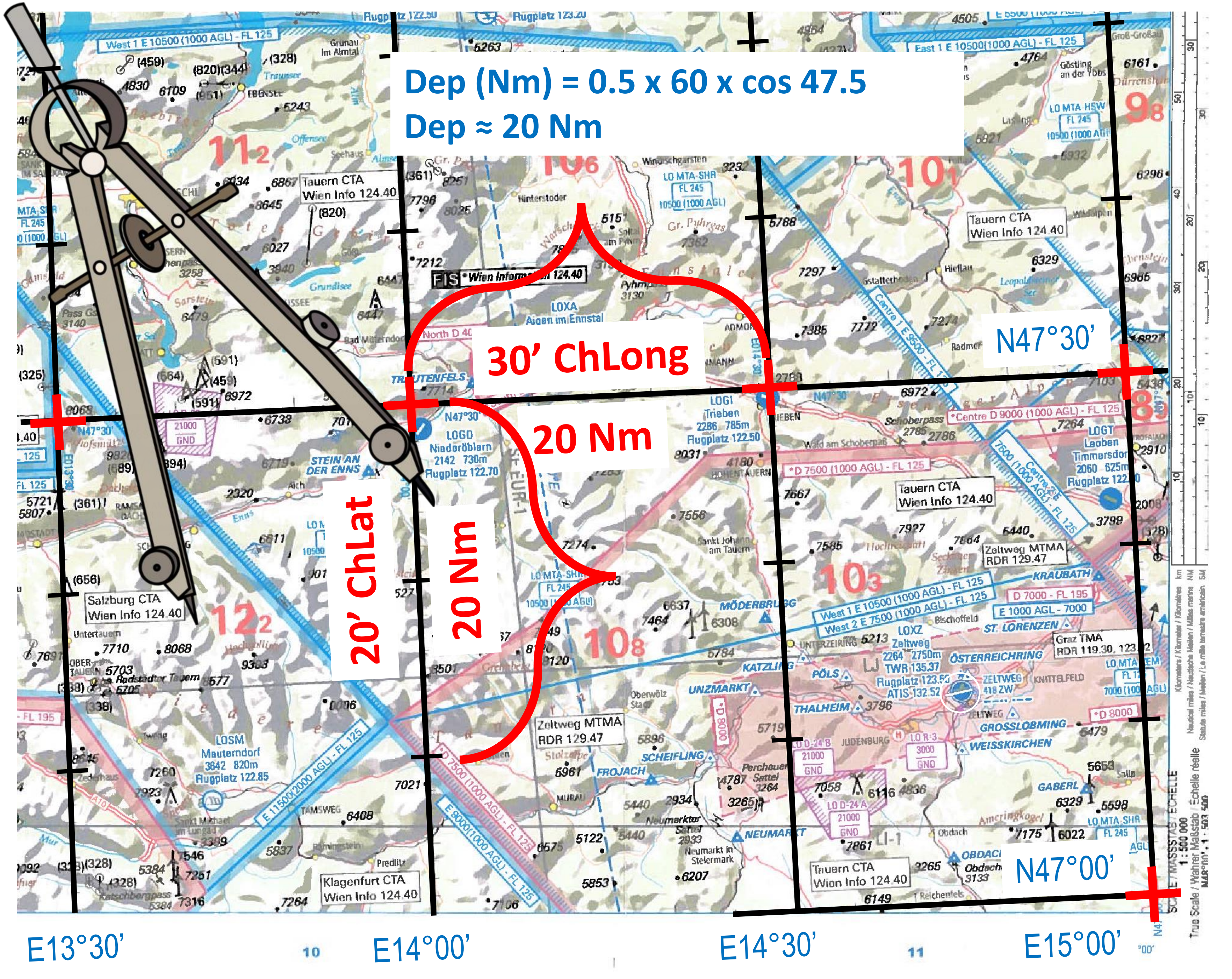
Then with a compass, if we translate this distance against a meridian, we can see that this gives us 20' change of latitudes, which is 20 Nm.

→ So the scale on this map is correct and the same everywhere.

→ Also the meridians and parallels cut at 90° angle.

This means that if we maintain 360° or 180° track, we remain on the meridians and go North or South, and if we maintain 090° or 270° track, we remain on the parallel and go East or West.

→ So this map is <u>conformal</u> or <u>orthomorphic</u> for navigation!



Scale

The scale of a map is the ratio of a distance on the map to the corresponding distance on the ground. The scale is mathematically written as the Representation Factor (RF) over the Earth Distance (ED)

$$\boldsymbol{Scale = \frac{RF}{ED}}$$

The common ICAO scale writing is 1:500.000, this means that 1 unit of distance on the map is 500.000 times that unit of distance on the Earth (1mm /500.000 mm; 1 SM/500.000 SM; etc.).
For convenience we use 1 cm on the map representing 500.000 cm on Earth or 5 km. To rapidly convert cm into km, only remove 5 zeros on the denominator number to read it in km.
Eg. *1/2500000 is 1 cm for 25 km*.

While on some maps, the unit used for the scale is mentioned
Eg. *1 inch ↔ 250 Nm*.

A scale is said to be big or small when compared to another scale. In this example we will compare Scale_A=1/250.000 and Scale_B=1/500.000

When dividing 1 by the denominator, Scale_A has a bigger number than Scale_B.

Scale_A=1/250.000 (ideal for low speed flight and search operation) **>**	**Scale_B**=1/500.000 (ideal for high speed flight and long navigation)
Zoom in	Zoom out
Less areas covered	**More areas** covered
More details can be seen	**Less details** can be seen

To obtain a scale, we can simply divide a known distance on Earth by the measured distance on the map in centimetre.

$$\boldsymbol{Scale = \frac{1}{\left(\frac{Known\ Distance\ on\ Earth}{measured\ distance\ on\ the\ map}\right)}}$$

To obtain the Earth distance from a measured distance on the map in cm, we can simply multiply measured distance on the map by the know distance on Earth for 1 cm

$$\boldsymbol{Earth\ Distance = measured\ distance\ on\ the\ map \times\ known\ dist\ on\ Earth}$$
$$\boldsymbol{=\ measured\ distance\ on\ the\ map\ \times \frac{1}{scale}}$$

To obtain the distance on the map in cm for a given Earth distance, we can simply divide the Earth Distance it by the know distance on Earth for 1 cm

$$\boldsymbol{Distance\ on\ the\ map = \frac{Given\ distance\ on\ Earth}{nown\ distance\ on\ Earth} =\ Given\ distance\ on\ Earth\ \times \frac{1}{scale}}$$

Exercise

a. On a chart a straight line is drawn between two points and has a length of 4.63 cm.
What is the chart scale if the line represents 150 NM?

b. A chart has the scale 1: 1 000 000. From A to B on the chart measures 3.8 cm, the distance from A to B in NM is:

c. At latitude 60°N the scale of a Mercator projection is 1:5 000 000.
The length on the chart between 'C' N60° E008° and 'D' N60° W008° is:

The correction is in the next page…

Exercise *(corrected)*

a. On a chart a straight line is drawn between two points and has a length of 4.63 cm.
What is the chart scale if the line represents 150 NM?

First we need set the same unit of distance on the map and on Earth:

- 150 Nm x 1.852 = 277.8 km
- 277.8 km / 4.63 cm = 60 km / cm

So 1 cm on the map represents 60 km on Earth, or 6.000.000 cm on Earth, so the scale is:

$$\boldsymbol{Scale = \frac{1}{6.000.000}}$$

b. A chart has the scale 1: 1 000 000. From A to B on the chart measures 3.8 cm, the distance from A to B in NM is:

- 3.8 cm x 1 000 000 = 3 800 000 cm or 38 km
- 38 km / 1.852 = 20,5 Nm

So 3.8 cm on this map represents **20,5 Nm** on Earth

c. At latitude 60°N the scale of a Mercator projection is 1:5 000 000.
The length on the chart between 'C' N60° E008° and 'D' N60° W008° is:

First we need to find the distance on Earth:

- Dep (Nm) = 16° x 60 x cos60° = 480 Nm

First we need to set the same unit of distance on the map and on Earth :

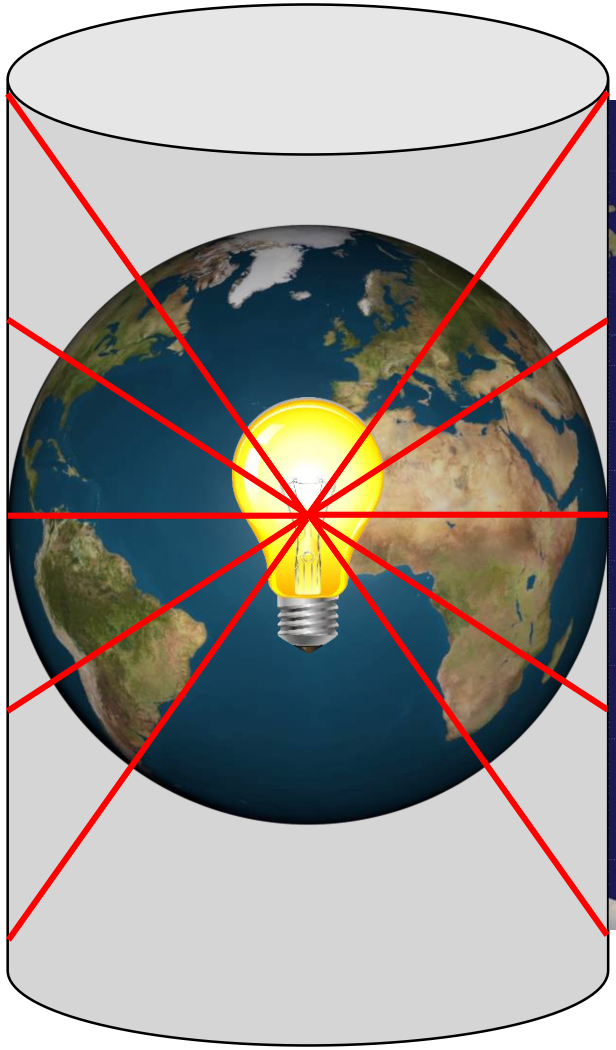
- 480 Nm x 1.852 = 888.96 km
- 1/5 000 000 means 1 cm on the map represents 50 km
- 888.96 km / 50 km.cm^{-1} = 17.8 cm

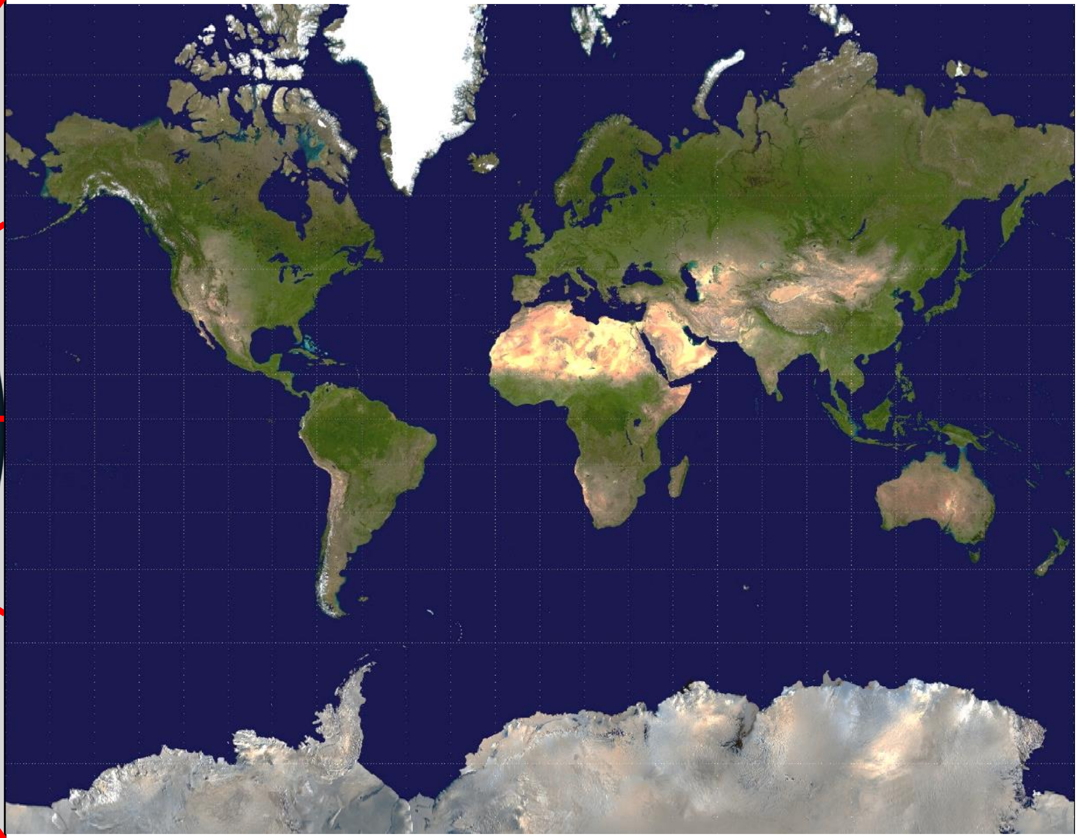
The length on the chart between C and D is **17.8 cm**

Mercator Projection

It is a cylindrical map projection presented by the Flemish geographer and cartographer Gerardus Mercator in 1569. The technique described above would produce a **perspective** projection. This **simple cylindrical** projection technique is illustrated below.

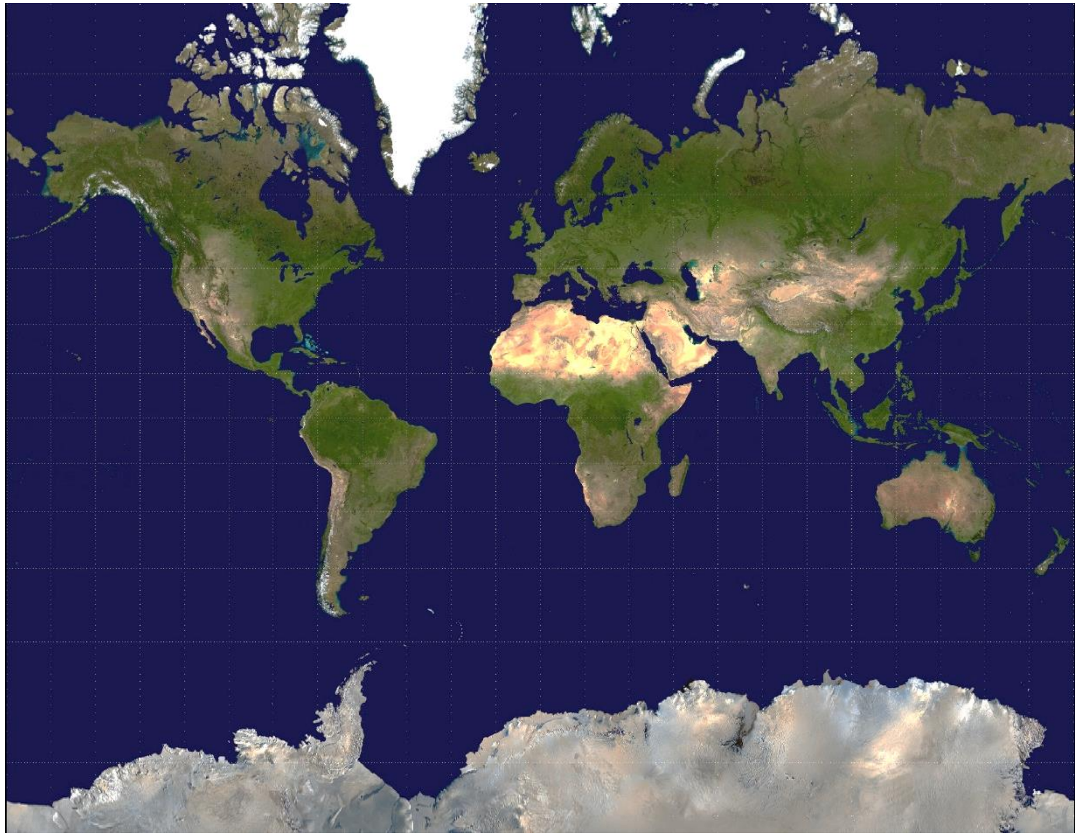
Gerardus Mercator (1512–1594)





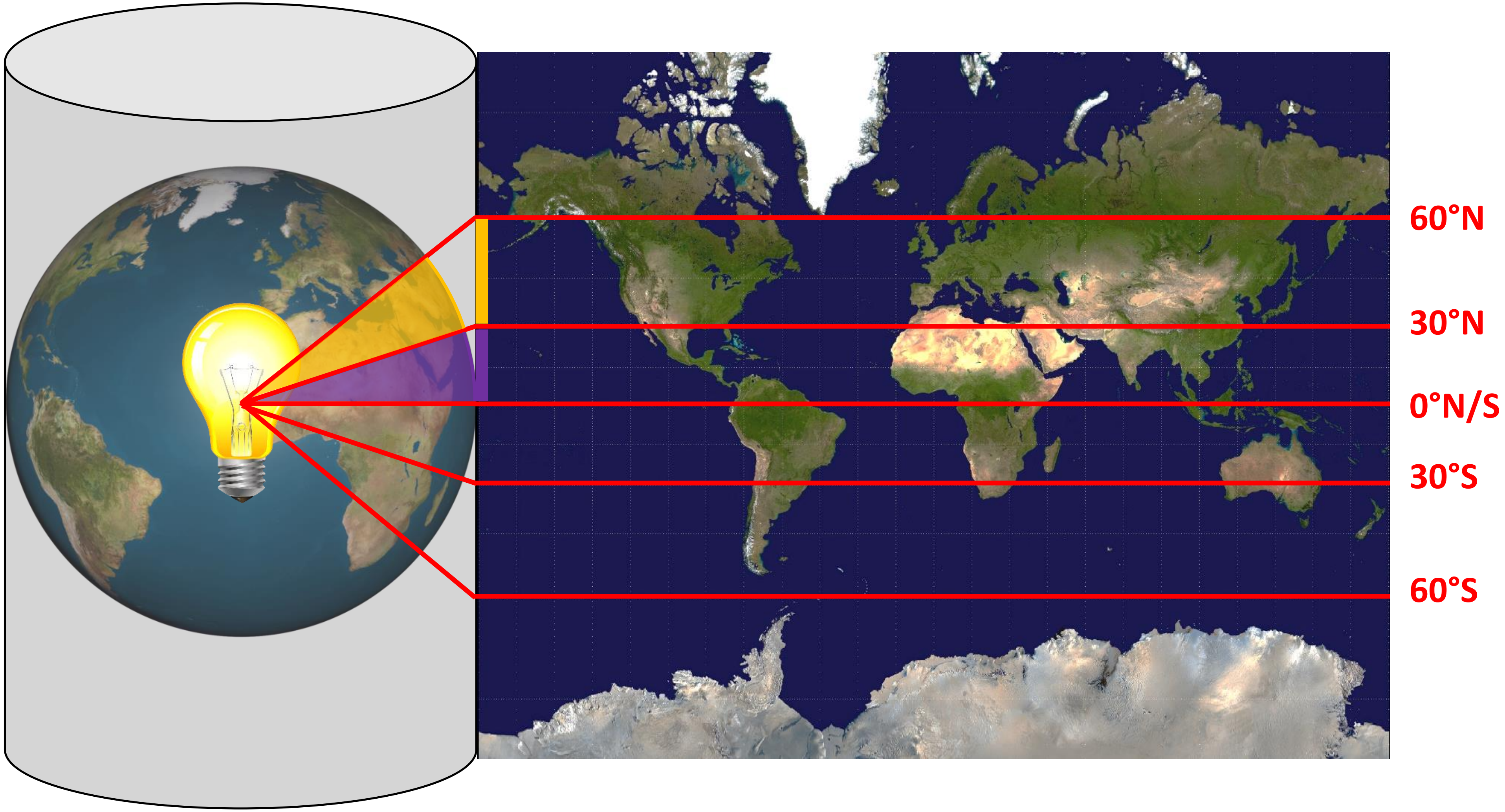
The earliest method of transferring the graticule of meridians and parallels from a globe to a flat sheet of paper was achieved using cylindrical projections. A scale model of the earth, the Reduced Earth (RE), was made at an appropriate scale. A cylinder of paper was wrapped around the RE, touching the RE at the Equator. Using a light source at the centre of the RE, the graticule was projected onto the cylinder. The cylinder was then ‘developed’ or opened up to a flat sheet of paper.

The projected graticule had one significant advantage for early navigators - **the meridians were equally spaced parallel lines**. Thus a straight line drawn on the chart would have a constant direction - **the straight line on the chart would be a Rhumb Line**. With their basic compass systems, early navigators preferred to sail constant directions and had to accept the fact that they would be sailing a Rhumb Line. Modern navigators use more advanced guidance systems and normally aim to fly the Great Circle track (more of this later).



In the 16th century, a Flemish navigator called Gerhard Kremer, who used the Latin alias ‘Mercator’, recognised the limitations of the simple cylindrical projection. The projected graticule met one of the requirements for an orthomorphic/conformal chart - the meridians and parallels crossed at right angles. Therefore, a straight line was a line of constant direction – a Rhumb line. Unfortunately, it was not the correct direction. The shapes were clearly not correct and therefore angles on the chart were not correct.

Note the shapes in Figure below - they are stretched in a N-S direction. Mercator realised that this was caused by failure to meet the second requirement of orthomorphism/conformality, namely that - **at any point on a chart, scale should be the same in all directions, or should change at the same rate in all directions**. On the simple cylindrical projection, the N-S scale was changing at a different rate from the E-W scale. Mercator determined that the E-W scale was changing such that, at any latitude, the scale was proportional to the secant of the latitude (secant = 1/cosine). However, the N-S scale was changing such that, at any latitude, the scale was proportional to the tangent of the latitude, resulting in the N-S stretching of shapes.



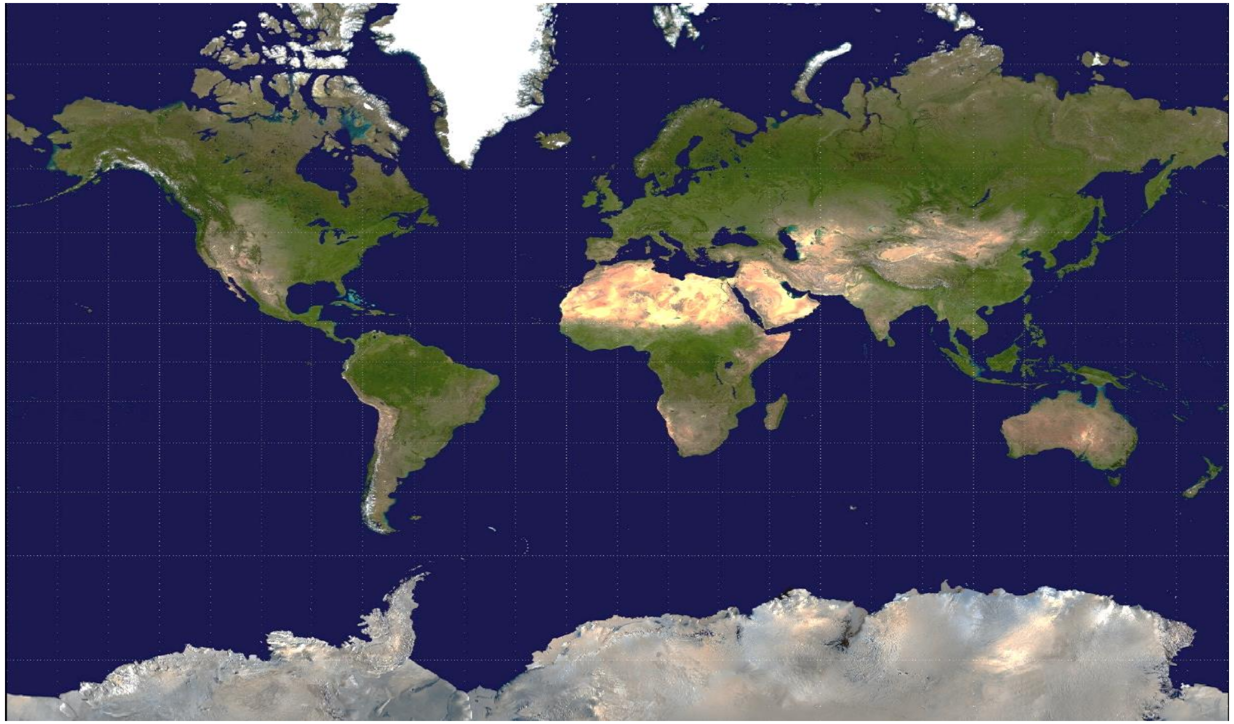
Mercator solved the problem by adjusting the positions of the parallels of latitude. The parallels had been projected as parallel lines with separation between the parallels increasing in proportion to the tangent of the latitude. Mercator adjusted the parallels of latitude so that their separation increased only at a rate proportional to the secant of latitude, matching the E-W scale change.

Briefly, Mercator mathematically adjusted the positions of the parallels of latitude to make the chart orthomorphic/conformal.

Because the chart has been mathematically produced, it is a **non-perspective** chart.

Mercator solved the problem in 1569. Because his solution was so simple, elegant, and correct, we still use his projection today, over 400 years later. A modern Mercator chart looks very different from his own because we have subsequently discovered and explored so many more countries. However, the basic principles of the graticule have not changed. He got it right.

An example of the adjusted Mercator projection is given in this Figure.



This Mercator projection is often called a **normal** or **direct** Mercator. The projection surface touches the Reduced Earth at the Equator. The geographic poles cannot be projected (they are on the axis of the cylinder).

Scale on Mercator

Mercator scale expands as the secant of the latitude.

This arises out of the departure formula. You will remember that:

Departure = change of longitude (min) × cos latitude

Or, Departure = distance at the equator × cos latitude

Mercator realised that, by the use of a cylindrical projection, the meridians were produced as parallel lines. This meant that a change of longitude on the chart was always represented by the same distance on the chart (so many centimetres), whatever the latitude. Therefore, we are holding the change of longitude (the Chart Length) constant and we need to see how the departure (the Earth Distance) changes with latitude. This tells us how the scale changes with latitude. We therefore re-arrange the formula as follows;

$$\textbf{Distance at any latitude } \boldsymbol{\lambda} = \textbf{Distance at Equator} \times \textbf{cos latitude } \boldsymbol{\lambda}$$

$$\frac{\mathbf{1}}{\textbf{Distance at any latitude } \boldsymbol{\lambda}} = \frac{\mathbf{1}}{\textbf{Distance at Equator}} \times \frac{\mathbf{1}}{\textbf{cos latitude } \boldsymbol{\lambda}}$$

$$\textbf{Scale at any latitude } \boldsymbol{\lambda} = \textbf{Scale at Equator} \times \frac{\mathbf{1}}{\textbf{cos latitude } \boldsymbol{\lambda}}$$

However, in mathematics there is a trigonometrical function called the secant. Its definition is that: $\textbf{secant } \theta = \dfrac{\mathbf{1}}{\textbf{cos } \theta}$

Therefore we can re-write the above equation as: $\textbf{Scale at any latitude } \boldsymbol{\lambda} = \textbf{Scale at Equator} \times \textbf{secant latitude } \boldsymbol{\lambda}$

Scale

Scale at any latitude λ = Scale at Equator × secant latitude λ

Let's see how this works with a practical example. Take a case where the scale at the Equator is 1/1,000,000. Now let's calculate what is the scale on a Mercator chart will be at 8°N (or S) latitude.

$$\frac{1}{\text{Distance at latitude } 8°} = \frac{1}{1\,000\,000} \times \frac{1}{\cos 8°}$$

There is no point to do the entire calculation of each fraction. The result expected to be found is $\frac{1}{number}$. So simply multiply the denominators and the result put is as the denominator in the scale of that latitude:

$$\frac{1}{\boxed{\text{Distance at latitude } 8°}} = \frac{1}{\boxed{1\,000\,000}} \times \frac{1}{\boxed{\cos 8°}} \rightarrow \frac{1}{\boxed{990\,268}} = \frac{1}{\boxed{1\,000\,000}} \times \frac{1}{\boxed{\cos 8°}}$$

$= \quad \times \qquad\qquad = \quad \times$

This shows that the scale at 8°N (or S) latitude is 99% of the scale at the Equator, or within 1% of correct scale.
This 1% figure is of importance in navigation. Up to a scale error of 1%, we can regard a chart as being of "constant scale", which means that you can measure distances using a ruler. Once the scale error increases to more than 1%, we must find distances either by calculation or by using the local latitude scale and measuring small distances at a time with a pair of dividers.

To summarise:

- Mercator scale is correct (same as the Reduced Earth) at the Equator.
- Mercator scale expands as the secant of the latitude.
- Mercator scale is within 1% up to 8° from the Equator.
- Mercator scale is within ½% up to 6° from the Equator.

Scale

Another way to find the scale at any latitude from another latitude different that the Equator:

$$\text{Scale at latitude A} = \text{Scale at Equator} \times \text{secant latitude A} \quad \leftrightarrow \quad \text{Scale at Equator} = \frac{\text{Scale at latitude A}}{\text{secant latitude A}}$$

$$\text{Scale any latitude B} = \text{Scale at Equator} \times \text{secant latitude B} \quad \leftrightarrow \quad \text{Scale at Equator} = \frac{\text{Scale at latitude B}}{\text{secant latitude B}}$$

$$\text{Scale at Equator} = \frac{\text{Scale at latitude A}}{\text{secant latitude A}} = \frac{\text{Scale at latitude B}}{\text{secant latitude B}} \quad \leftrightarrow \quad \frac{\frac{\cancel{1}}{\text{Distance at latitude A}}}{\frac{\cancel{1}}{\text{cos latitude A}}} = \frac{\frac{\cancel{1}}{\text{Distance at latitude B}}}{\frac{\cancel{1}}{\text{cos latitude B}}}$$

$$\leftrightarrow \quad \boxed{\frac{\text{Distance at latitude A}}{\text{cos latitude A}} = \frac{\text{Distance at latitude B}}{\text{cos latitude B}}}$$

Eg. On a direct Mercator Projection, the scale at 30°N is 1:800 000, what is the scale at 45°S on this map?

$$\frac{\text{Distance at latitude 30°}}{\cos 30°} = \frac{\text{Distance at latitude 45°}}{\cos 45°} \quad \leftrightarrow \quad \text{Distance at latitude 45°} = \text{Distance at latitude 30°} \times \frac{\cos 45°}{\cos 30°}$$

$$\text{Distance at latitude 45°} = \mathbf{653\ 197}$$

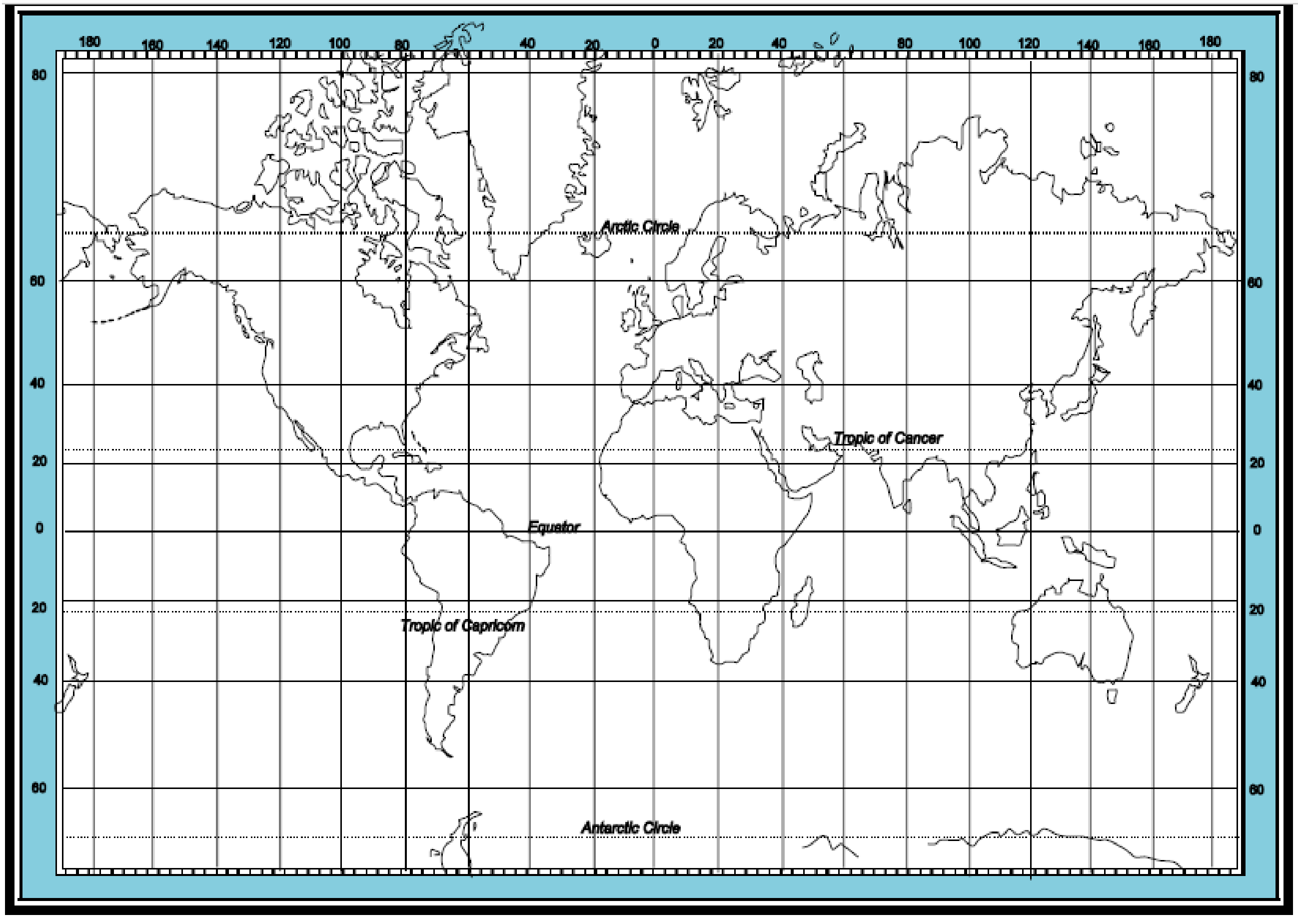
→ The scale at 45°S is **1:653 197**

Orthomorphism

All charts used for navigation must be orthormorphic. This chart is **orthomorphic/conformal** by mathematical construction (Mercator's adjustment of the parallels of latitude). The projection is **non-perspective**.

Graticule

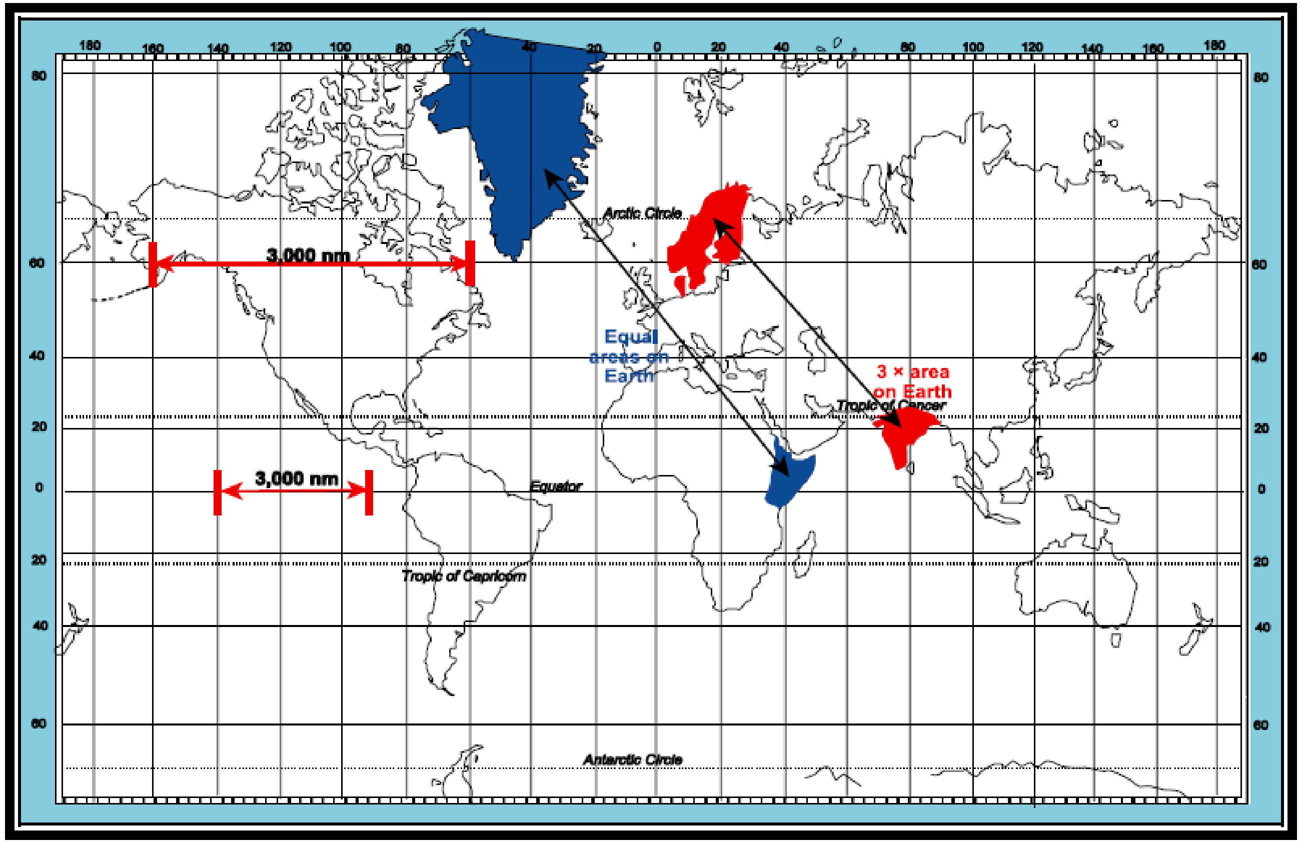
The graticule is rectangular. **Meridians are equally spaced parallel lines. Parallels of latitude are parallel lines with the space between them increasing as the secant of the latitude.**



Shapes

Mercator produced an orthomorphic projection. However, looking again at Figure below, you should notice that the areas are not correctly represented. Correct representation of area is not required on an orthomorphic chart.

Thus, Greenland **appears as large as Africa** on the chart, despite the fact that the land area of Africa is approximately 18 times that of Greenland. In reality, **the land area of Greenland is only the same as the small NE corner of Africa**, as illustrated in Figure below.



Similarly, the land area of Scandinavia illustrated is only one third of the land area of India, but on the chart they **appear** to have similar areas. Note also the chart length equivalent to 3000 nm at 60°N, which is twice the chart length for the same distance at the Equator.

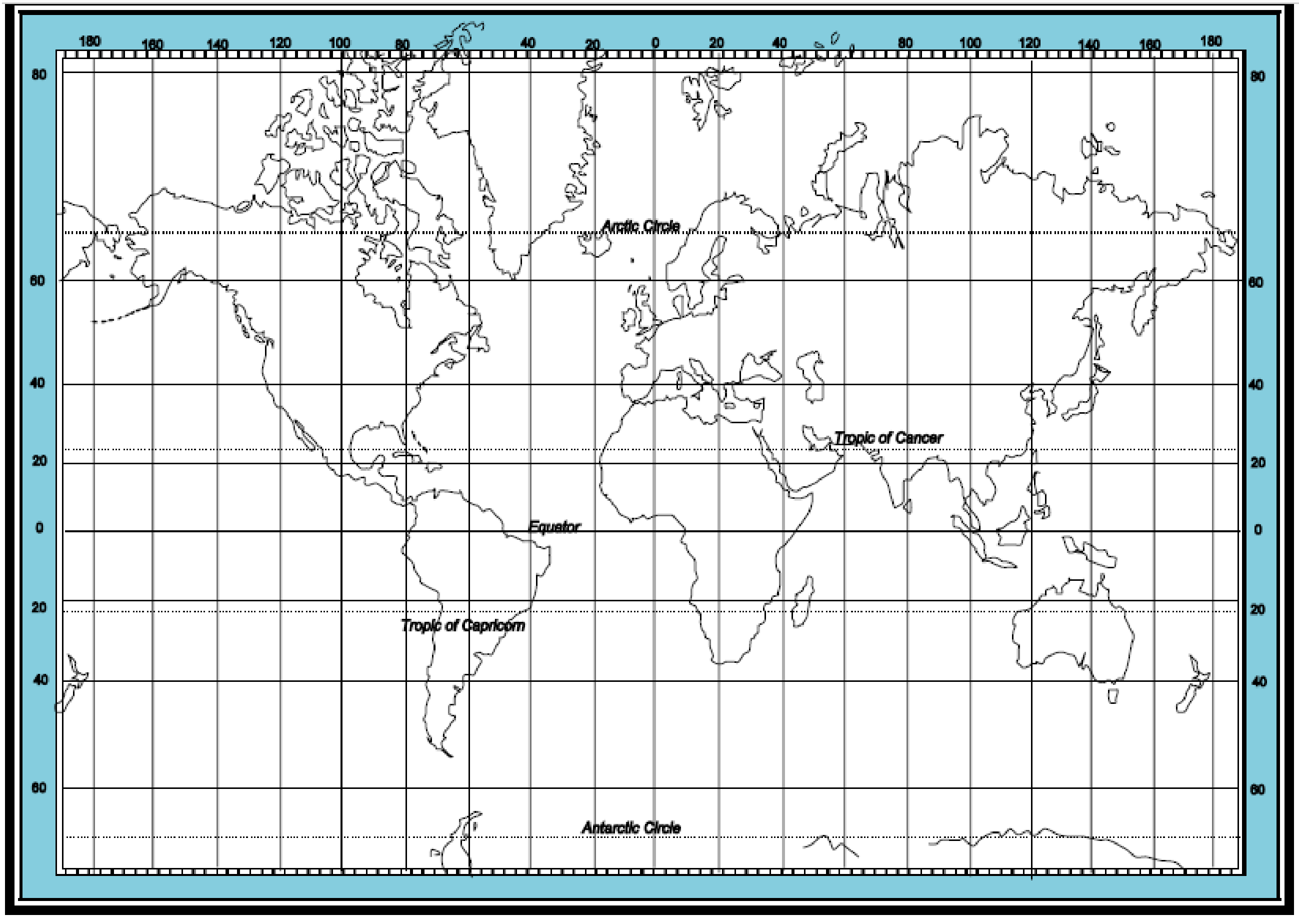
This distortion of area and change of scale also leads to change of shape. Land masses at high latitudes appear too wide too wide for their height compared with the same land masses on a globe or on a chart with convergent meridians.

These distortions of shape are insignificant over small distances and therefore have no effect on a pilot's ability to map-read. They have no implications for Rhumb Line navigation at all, but they can give a false impression of the most direct routing, especially at high latitudes. This point was covered in earlier.

Chart Convergence

The concept of **Earth Convergence**, or Convergency, was covered in **Earth Convergence**. It is the angle of inclination of the meridians **on the Earth**, (or the **change in direction of a Great Circle**), between 2 longitudes. However, for each type of projection, we also have the concept of **Chart Convergence**. This is the angle of inclination between meridians **on the chart**, (or **the change in direction of a straight line**), between 2 longitudes.

For a Mercator chart, as all meridians are parallel, their mutual inclination is **zero**. The change in direction of a straight line drawn on the map is also **zero**. It will always cut all meridians at the same angle. That is why Mercator produced the projection in the first place - so that a straight line on the chart gives a single track angle.



Earth convergency is also zero at the Equator, but nowhere else. Therefore Mercator convergence is correct at the Equator (only) but constant everywhere (always zero).

To calculate the Convergency on Mercator, we need to use the Earth Convergence formula:

Chart Convergence = ChLONG x sin Mean LAT

Rhumb Lines

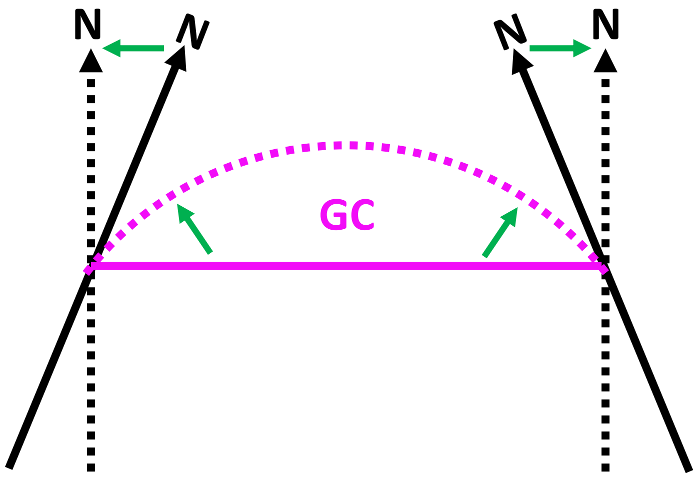
Because the meridians are parallel lines, a straight line track drawn on the chart will cut all meridians at the same angle. **A Straight Line track on a Mercator chart is a Rhumb Line**.

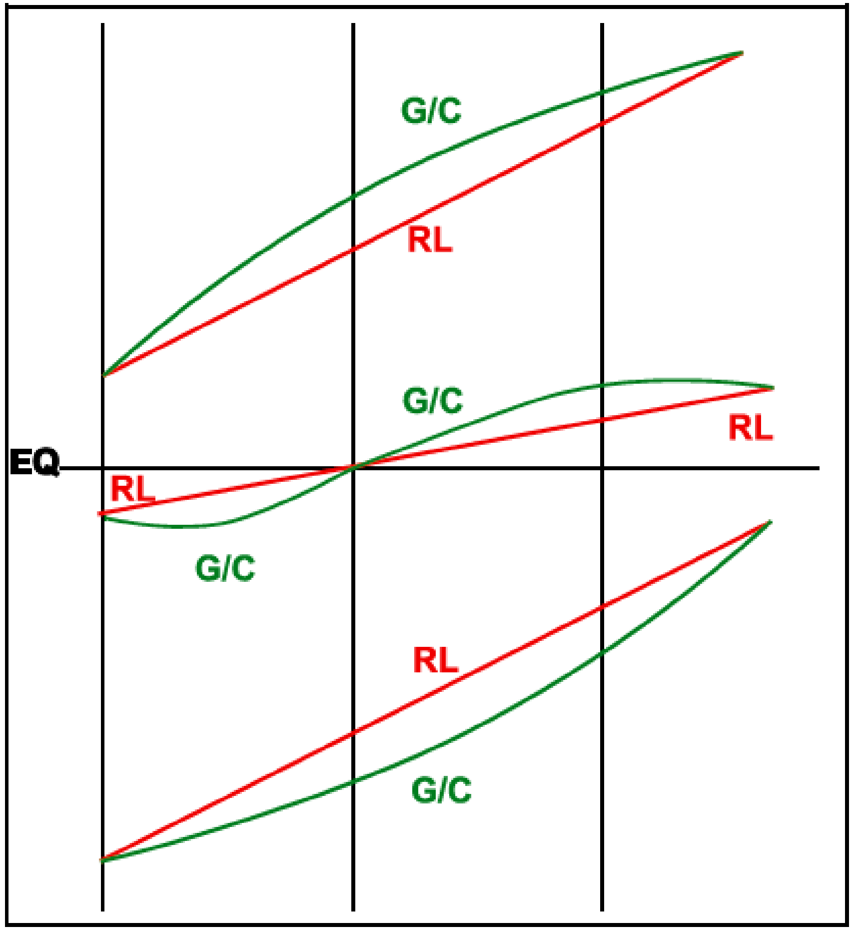
Great Circles

The Rhumb line between two points will always be nearer to the Equator than the corresponding Great Circle. Conversely, the Great Circle between two points will always lie nearer the Pole than the Rhumb line.

This was covered in detail before. The statement can be simplified in this Figure.

The Mercator projection removes the convegrency of the meridians, which makes **the Great Circles to concave to the Equator**.

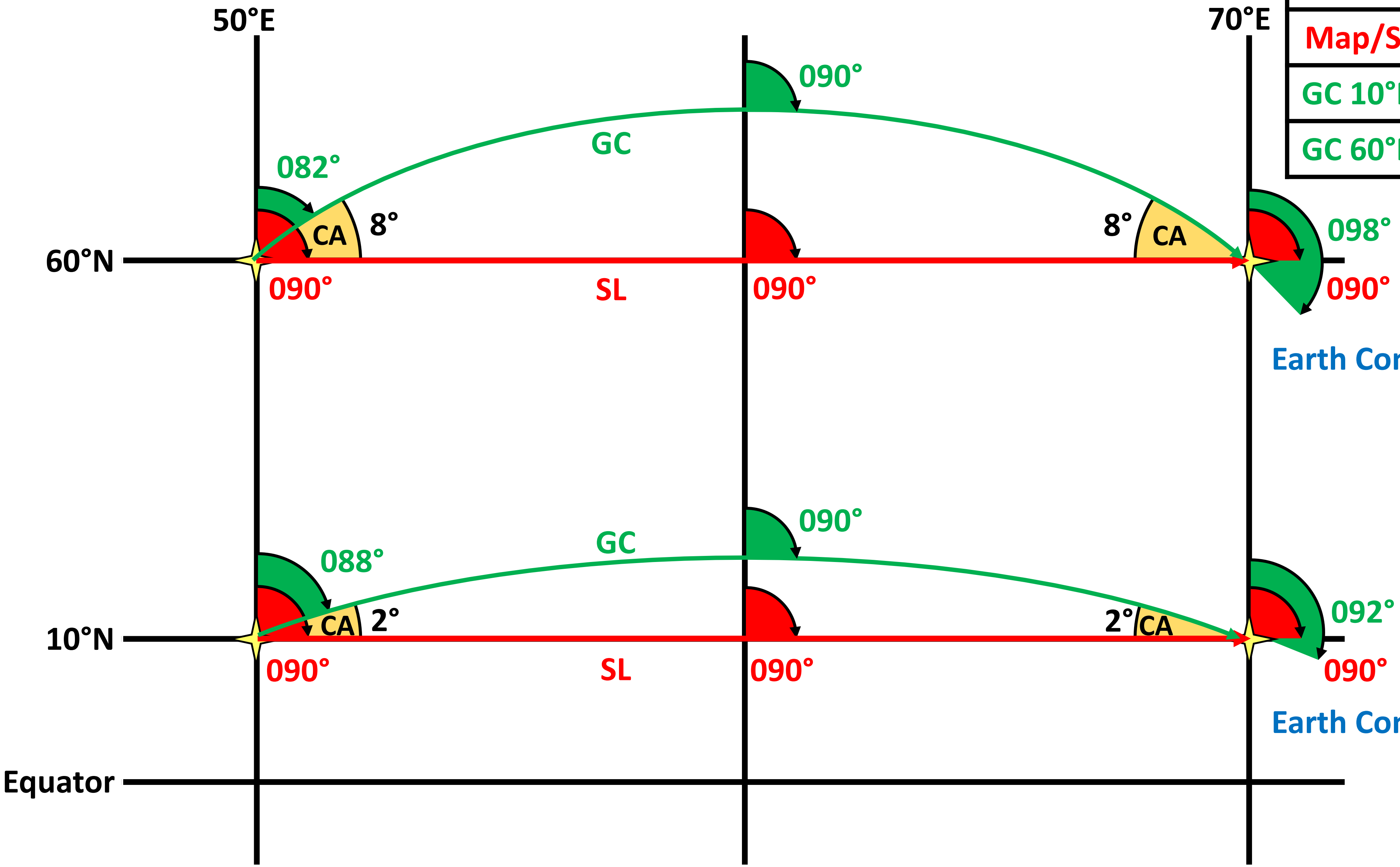




When a navigation is planned, we simply draw a straight line between two points and we follow the Straight Line.

Let's see how close is a **Straight Line (SL)** to the **Great Circle (GC)** at latitude 10°(N/S) and at latitude 60°(N/S) in the next slide.

Track	Initial	Halfway	Final
Map/SL	090°	090°	090°
GC 10°N	088°	090°	092°
GC 60°N	082°	090°	098°



Earth Convergence = 20° x sin60° ≈ 16°

So Conversion Angle = 8°

Earth Convergence = 20° x sin10° ≈ 4°

So Conversion Angle = 2°

On Mercator Projection, following a **Straight Line (Rhumb Line)** near the Equator (max 10°N/S), the Track approximates a **Great Circle**, however at higher latitude, the Straight Line is far from the Great Circle and so the distance navigated will be much bigger.

SUMMARY OF MERCATOR PROPERTIES

A summary of Mercator properties is set out in the table. All of these are asked in examinations and they should be learnt.

PROPERTIES OF A MERCATOR CHART	
Scale	Correct on the Equator. Elsewhere increases as the secant of the latitude. Within 1% up to 8° from the Equator. Within ½% up to 6° from the Equator.
Orthomorphic	Yes. All charts used for navigation must be.
Graticule	Meridians are straight parallel lines, evenly spaced. Parallels are straight parallel lines with the space between them increasing with the secant of the latitude.
Shapes	Reasonably correct over small areas. Distortion over large areas, especially at high latitudes.
Chart Convergence	Zero everywhere. Correct at the Equator. Constant across the chart.
Rhumb lines	Straight lines Always! Everywhere!
Great Circles	Equator and meridians are straight lines (because they are also Rhumb Lines). All other Great Circles - curves, with a track nearer the Pole (or concave to the Equator).

Lambert Projection

We have seen that the Mercator chart has many powerful properties, but it does have 2 limitations. These are:

- Great Circles are not projected as straight lines.
- The chart is not constant scale. Indeed, scale changes quite rapidly on a Mercator chart.

Navigators wished to maintain Rhumb Line tracks for the first 400 years or so or the life of the Mercator chart, because they were steering by compass, and therefore needed to have a constant track direction.

However, that situation started to change from about 1960 onwards, when automatic computing became available. It is possible to calculate a desired Great Circle track direction by spherical trigonometry formulae, and the computers built into INS, IRS, FMS and GPS do this as a matter of course.

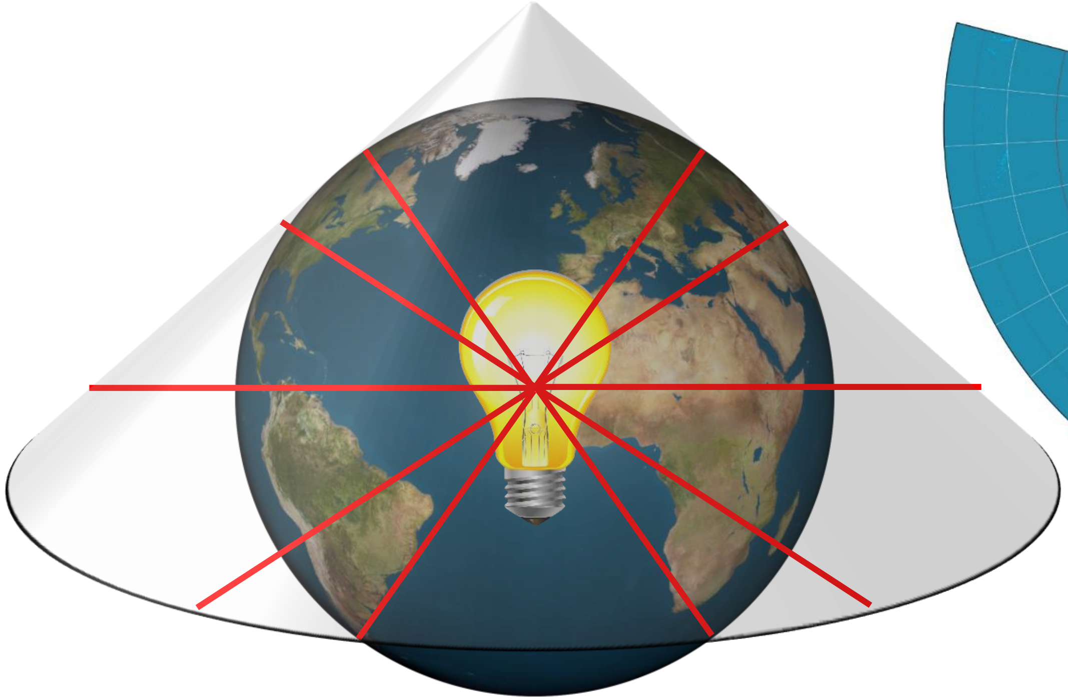
If the aircraft is going to be steered along a Great Circle, it would be helpful to have a chart on which a Great Circle is a straight line. Otherwise the aircraft will appear to go off track in the middle of the leg, then recover back to track again.

If the scale were to remain constant on a chart, we could measure distances with a ruler instead of having to use a pair of dividers, and have to open the dividers to different distances at different latitudes.

Lambert conformal conic projection (LCC)

It is a conic map projection used for aeronautical charts, is one of seven projections introduced by the Swiss mathematician and philosopher Johann Heinrich Lambert (1728-1777) in his 1772 publication.

Johann Heinrich Lambert (1728–1777)



A cone is placed over a reduced earth, in such a way that the cone is tangential with the reduced earth along a parallel of latitude.

On this map, since the paper in tangent to the mid latitude, on that latitude the shape is correct, however further away, the graticule and the areas are expanding, making this map only good to be used at the mid-latitudes.

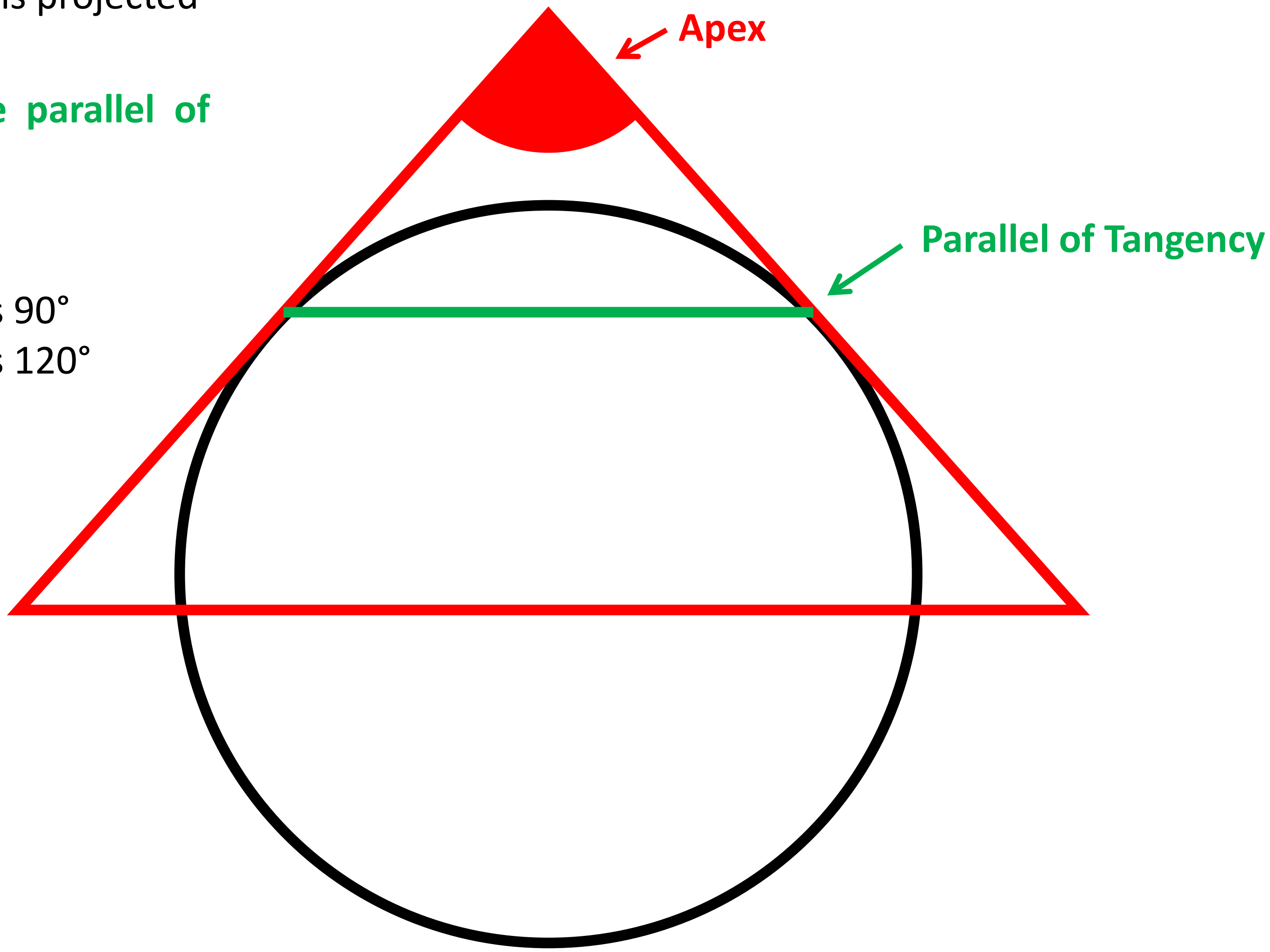
Notice that the meridians are straight lines, originating from the pole and they have the same North.

The **Apex of the cone** depends on the **parallel of tangency**, in other words, it depends on the latitude at the map is projected

The **angle** of the **apex** is twice the value of **the parallel of tangency.**

Eg.

- If the map is projected at 45°N, the apex angle is 90°
- If the map is projected at 60°N, the apex angle is 120°
- Etc.

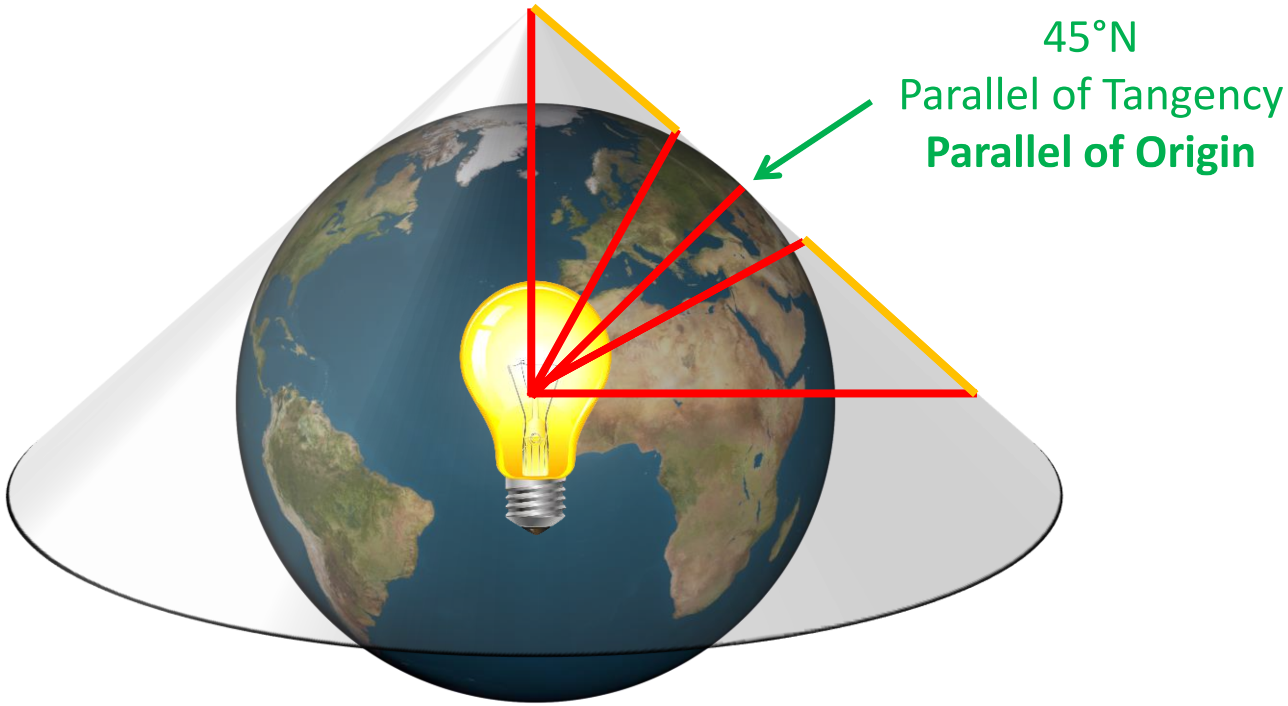


The Simple Conic Projection

The parallel of tangency is also called the **parallel of Origin** (**//o**)

The problem of the simple conic projection is the scale. Indeed, outside of the **parallel of origin**, the scale **expands** too rapidly.

This makes the projection not orthomorphic for navigation. So a technic shall be made to maintain the scale constant or to expand at constant rate.

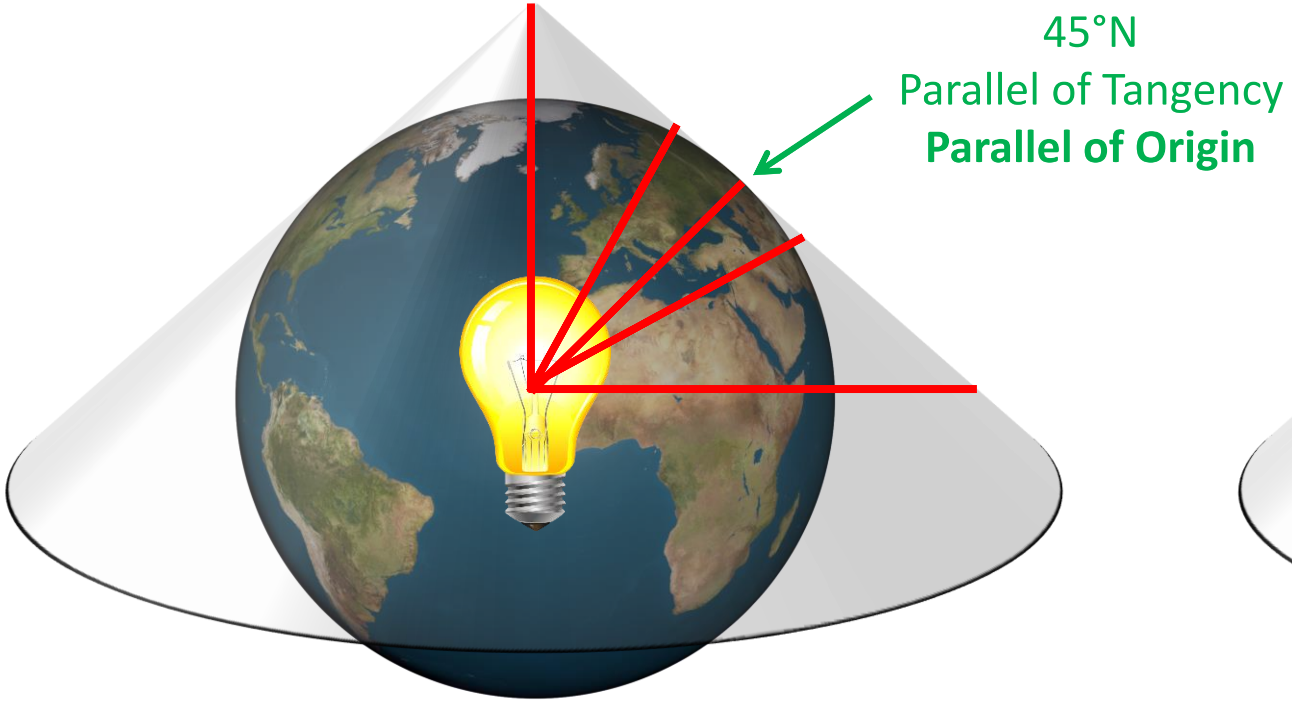


The Conical Orthomorphic Projection

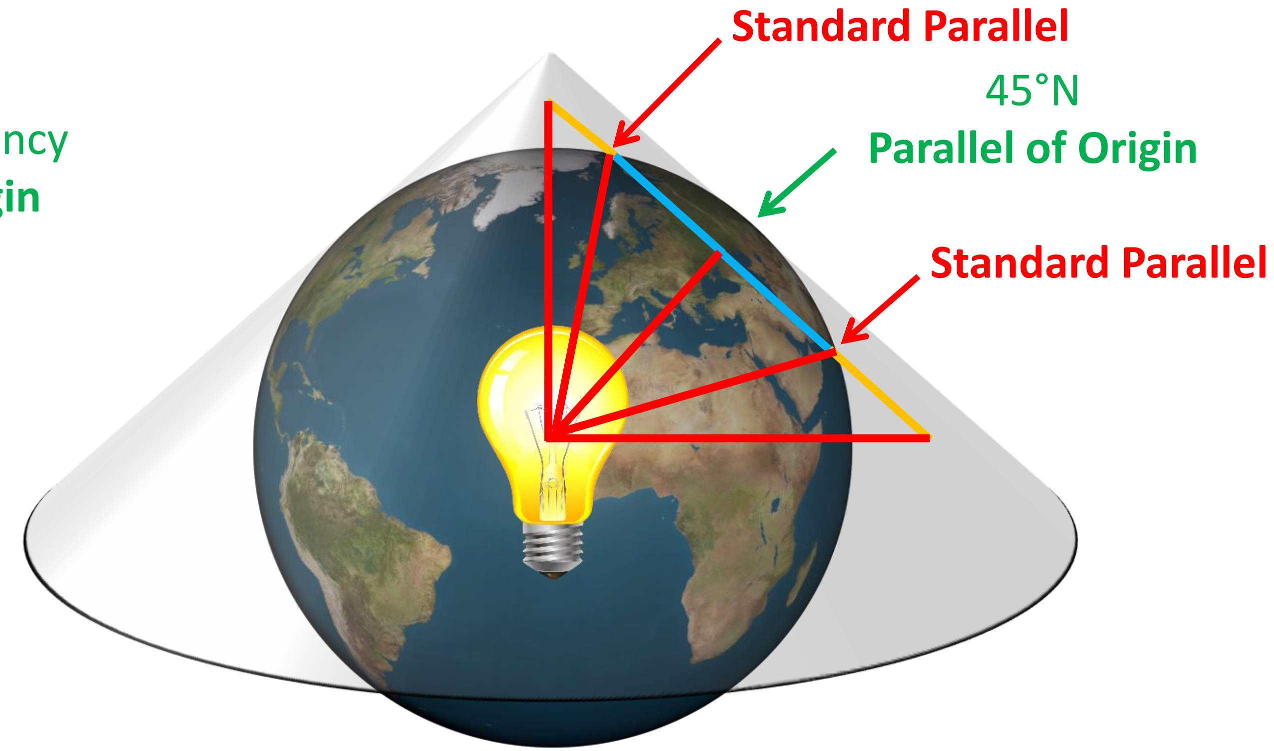
To maintain a slow expansion of the scale and satisfy orthomorphism, the projection is made with the cone crossing the reduced Earth. This is done pushing the cone inside of the globe or by reducing its dimensions by the same factor.

This projection is called **Lambert's Conical Orthomorphic Projection**, where the cones crosses the globe at the parallels, called the **Standard Parallels** (**STD//**), where the **parallel of origin** is assumed the be between the two standard parallels.

Eg. If the **Standard Parallels** (**STD//**) are **25°N** and **65°N**, the **parallel of origin** is **45°N**.



Simple Conical Projection



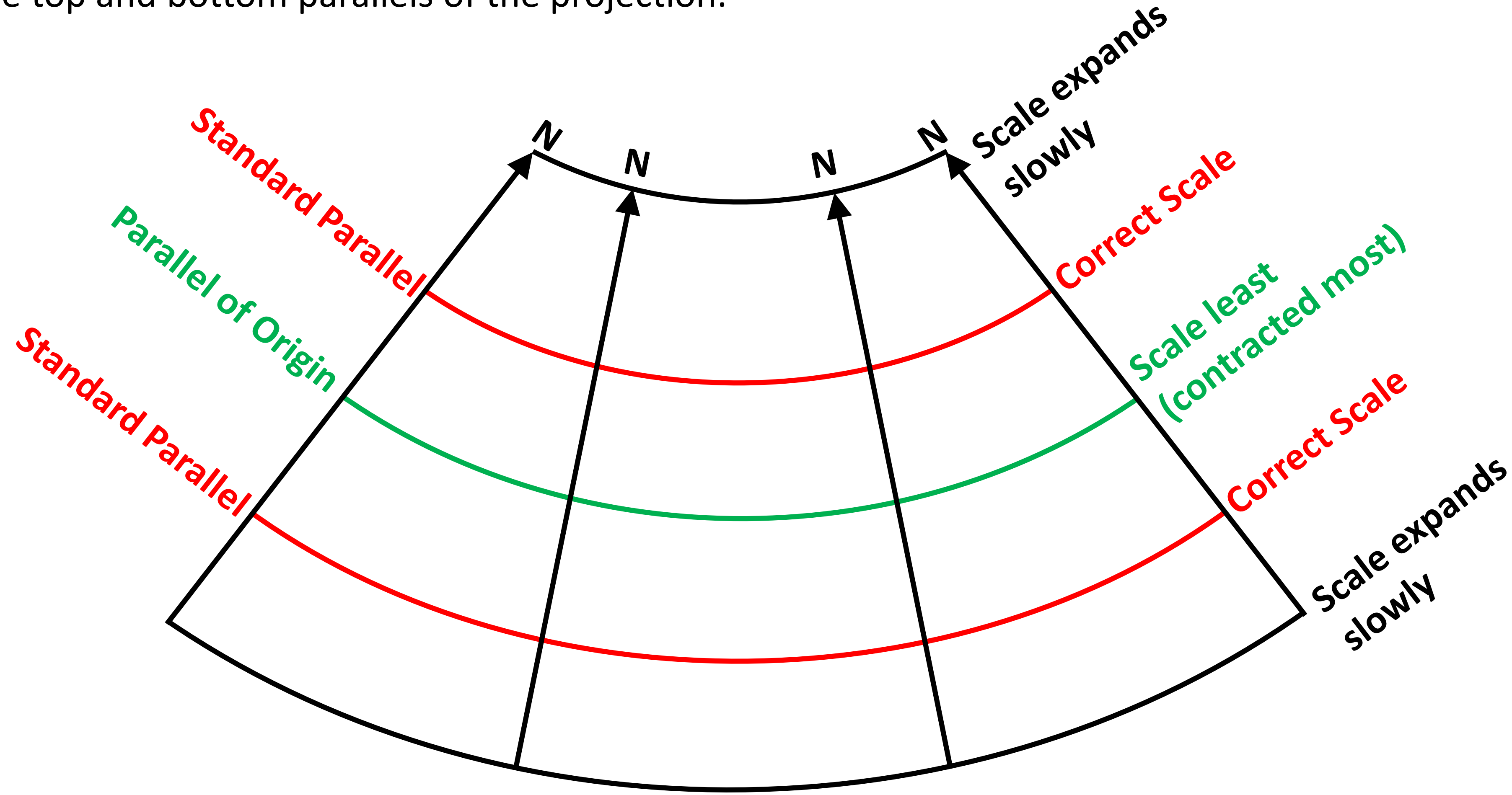
Lambert's Conical Orthomorphic Projection

Notice that the scale is **correct** at the **Standard Parallels**, and it **contracts** to the **parallel of origin**, then, the scale slowly **expands** outside of the **Standard Parallels**. Which keeps a slow change of the scale over a large change of latitudes.

Having brought the cone inside the Reduced Earth, it was necessary to make some mathematical adjustments in order to make the chart orthomorphic. The Lambert projection is a **non-perspective chart**.

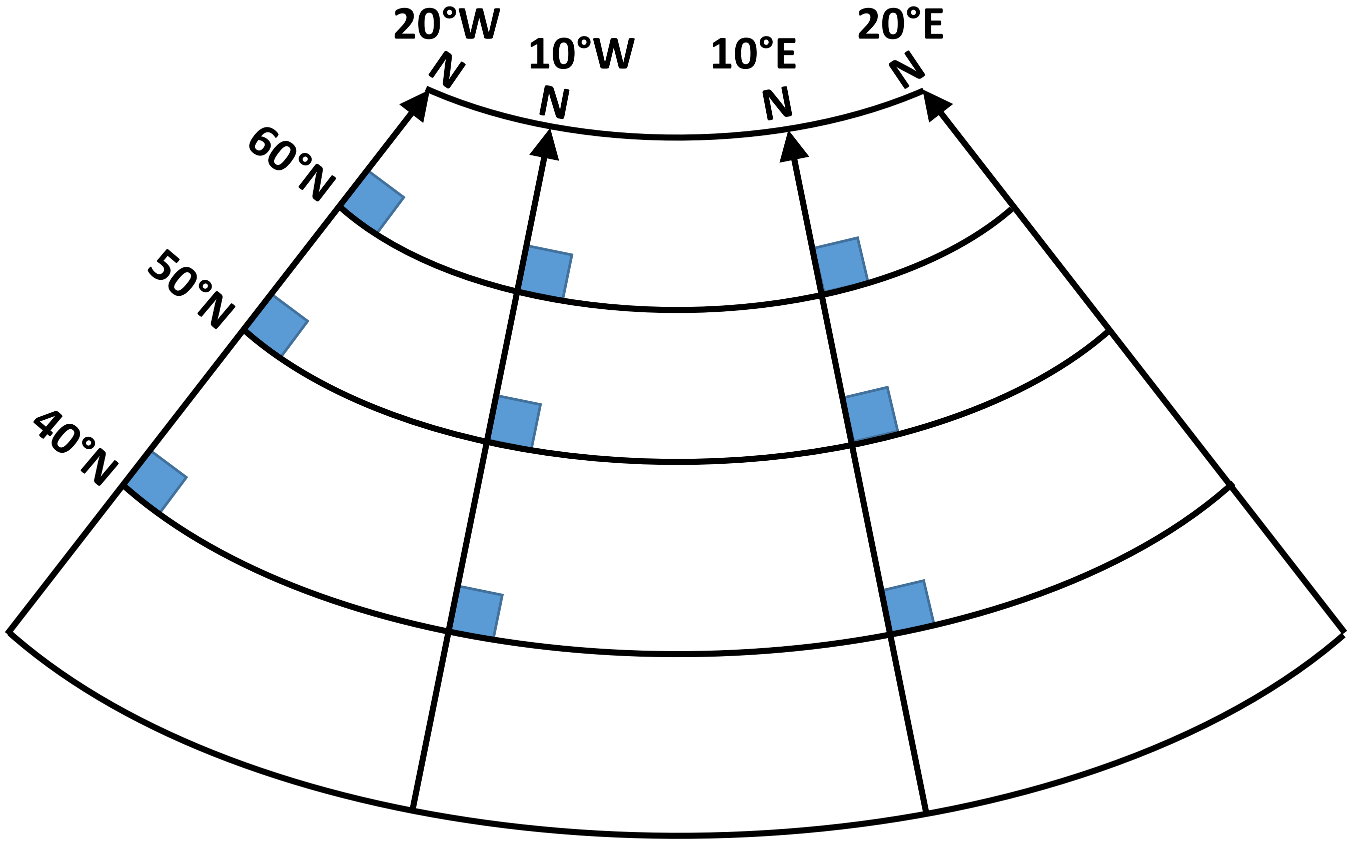
Scale

This is least on the Parallel of Origin. It expands away from the Parallel of Origin, until it is correct on the Standard Parallels. Scale is greatest on the top and bottom parallels of the projection.

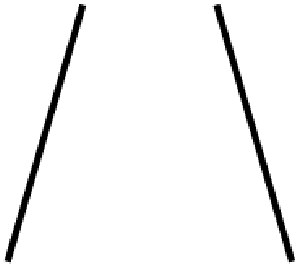


Orthomorphism

The chart is orthomorphic by mathematical construction.

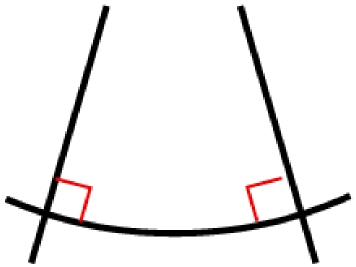


The meridians converge:



But the parallels curve:

When you put them together,
they cut at right angles.



Graticule

Meridians are straight lines radiating from the pole. Parallels of latitude are arcs of circles, all of which are centred at the pole. The pole is usually off the map sheet which you are using. The map sheet is shown as a red broken rectangle here.

Be aware of the map shape for the Conical Orthomorphic Projection in the Southern Hemisphere

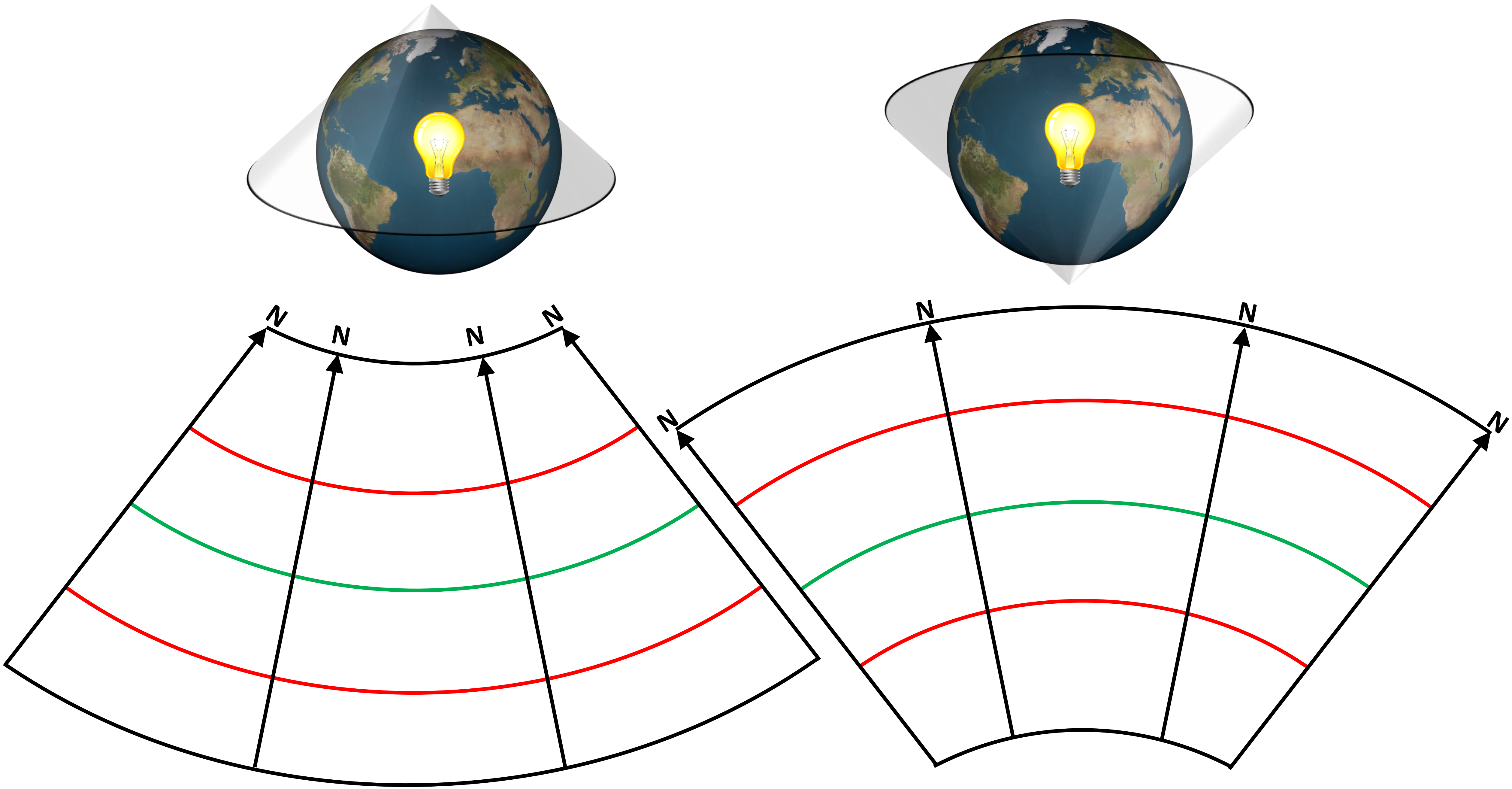
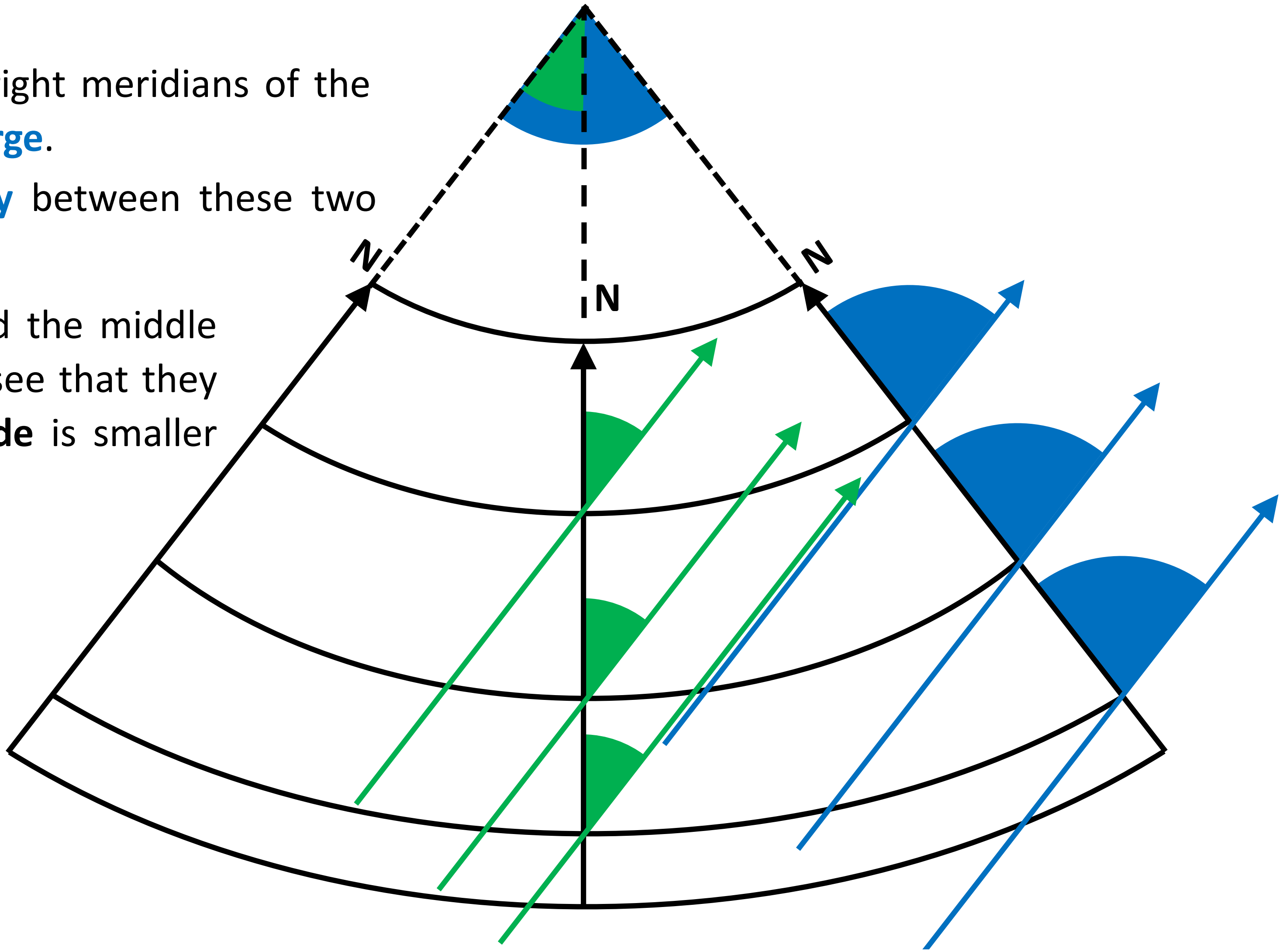


Chart Convergence

If we take we compare the most the left and the right meridians of the chart at a given latitude, we can see that they **converge**.

However we notice that the **angle of convergency** between these two meridians is the same at any latitude.



Now if we take we compare the most the left and the middle meridians of the chart at a given latitude, we can see that they **converge** less. Meanwhile their **change of longitude** is smaller than the left and right meridians.

However we notice that the **angle of convergency** between these two meridians is the same at any latitude.

So the chart convergency on Lambert Projection depends on the **change of longitude** and a **convergency factor 'n'**

Chart Convergency = Chlong x 'n'

Indeed on this chart, two meridians convergence angle is constant at any latitude, this is because the cone shape is constant. So two meridians will converge according to the cone apex, and since the cone apex depends on the chosen parallel of origin, it means that the meridians converge by the convergency of the parallel of tangency, so the Parallel of Origin. So on a Lambert projection, the chart convergency is:

Chart Convergency = ChLONG x sin Parallel of Origin

Chart Convergence

We saw that the chart convergence depends on the Parallel of Origin, which is the parallel of tangency. However for a Lambert's Conical Orthomorphic Projection, the parallel of tangency are the Standard Parallels. Does this change the chart convegency?

In fact the chart convergence depends on the cone's apex angle which depends on the Parallel of Origin. In Lambert's Conical Orthomorphic Projection, the cone's apex angle doesn't change, simply the cone is pushed in the reduced Earth.

So since the cone shape doesn't change, so the meridians on the chart keep the same convergence. So the chart convergence is the same on a simple conic projection and on the Lambert's Conical Orthomorphic Projection.

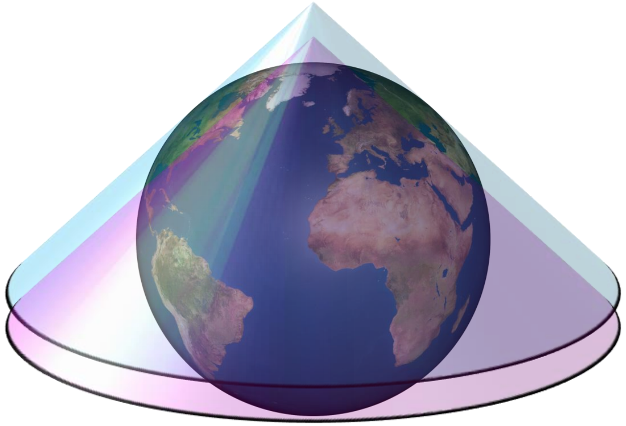


Chart Convergency = ChLONGx sin Parallel of Origin

The convergency factor on a Lambert projection depends on the cone's shape (Parallel of Origin) and it is constant across the chart, we call this factor the **constant of the cone 'n'**, and is simple to calculate it:

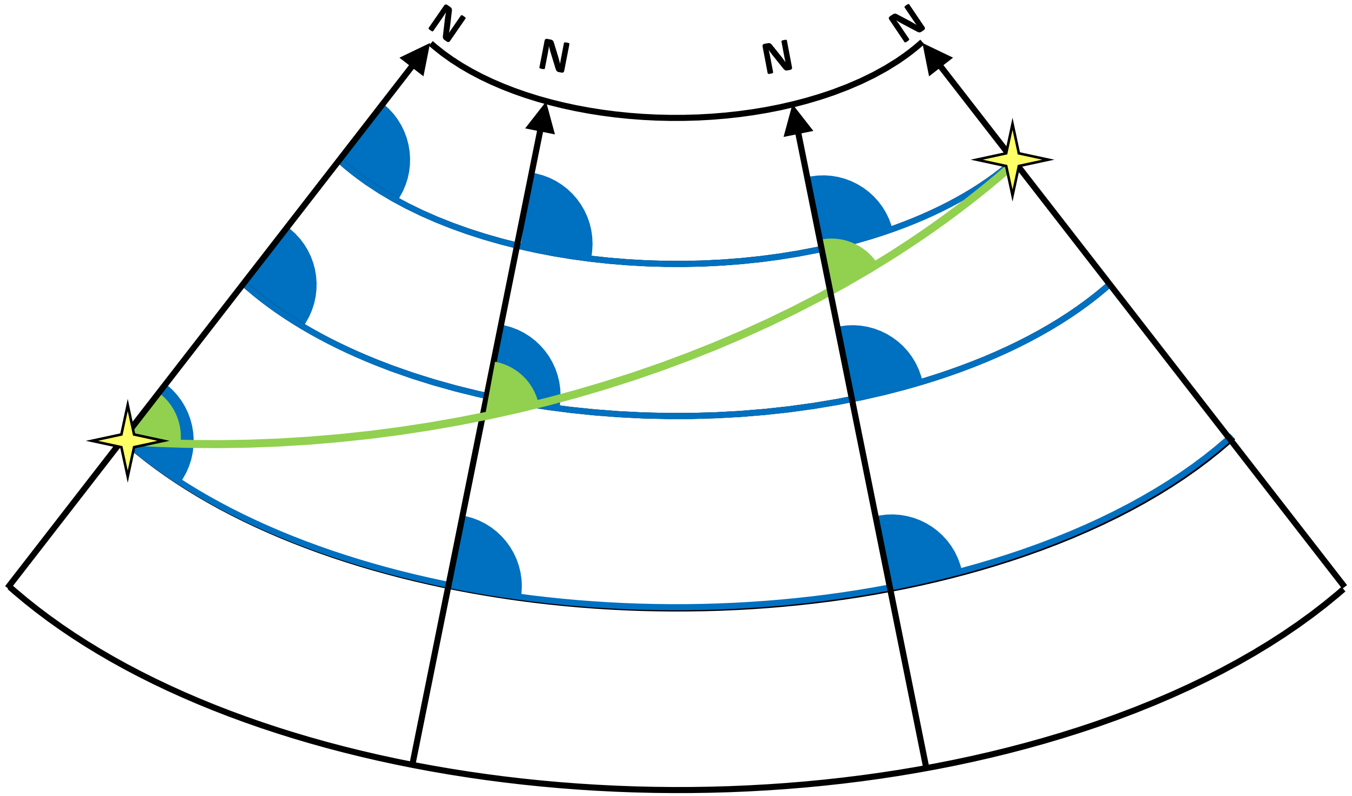
Constant of the cone 'n' = sin Parallel of Origin

We can re-write the equation of the chart convergence as following:

Chart Convergency = ChLONG x 'n'

Rhumb Lines

Except for meridians which appear as straight lines, Rhumb Lines are curves concave to the pole of the projection.



Great Circles

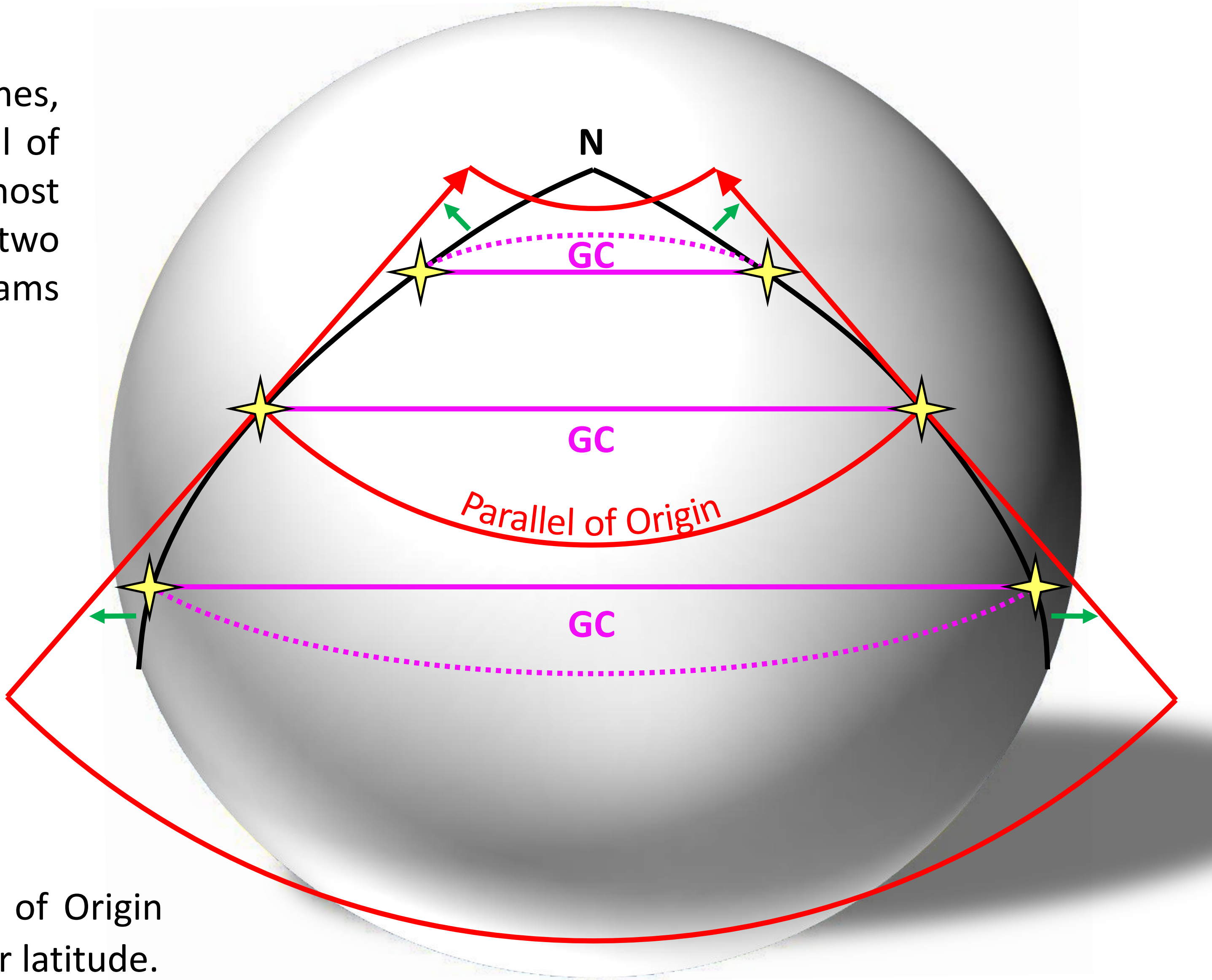
Except for the meridians which appear as Straight Lines, **Great Circles** appear as curves concave to the Parallel of Origin. A Straight Line in an 'East-West' direction most nearly represents a great circle when drawn between two positions on the parallel of origin. The following diagrams explain why:

The meridians converge towards each other with increasing latitude.

When we project this real Earth situation onto a Lambert chart.

The Earth meridians are straightened out into straight-line chart meridians. This means that the formerly straight-line Great Circles are stretched outwards at latitudes higher or lower than the parallel of origin.

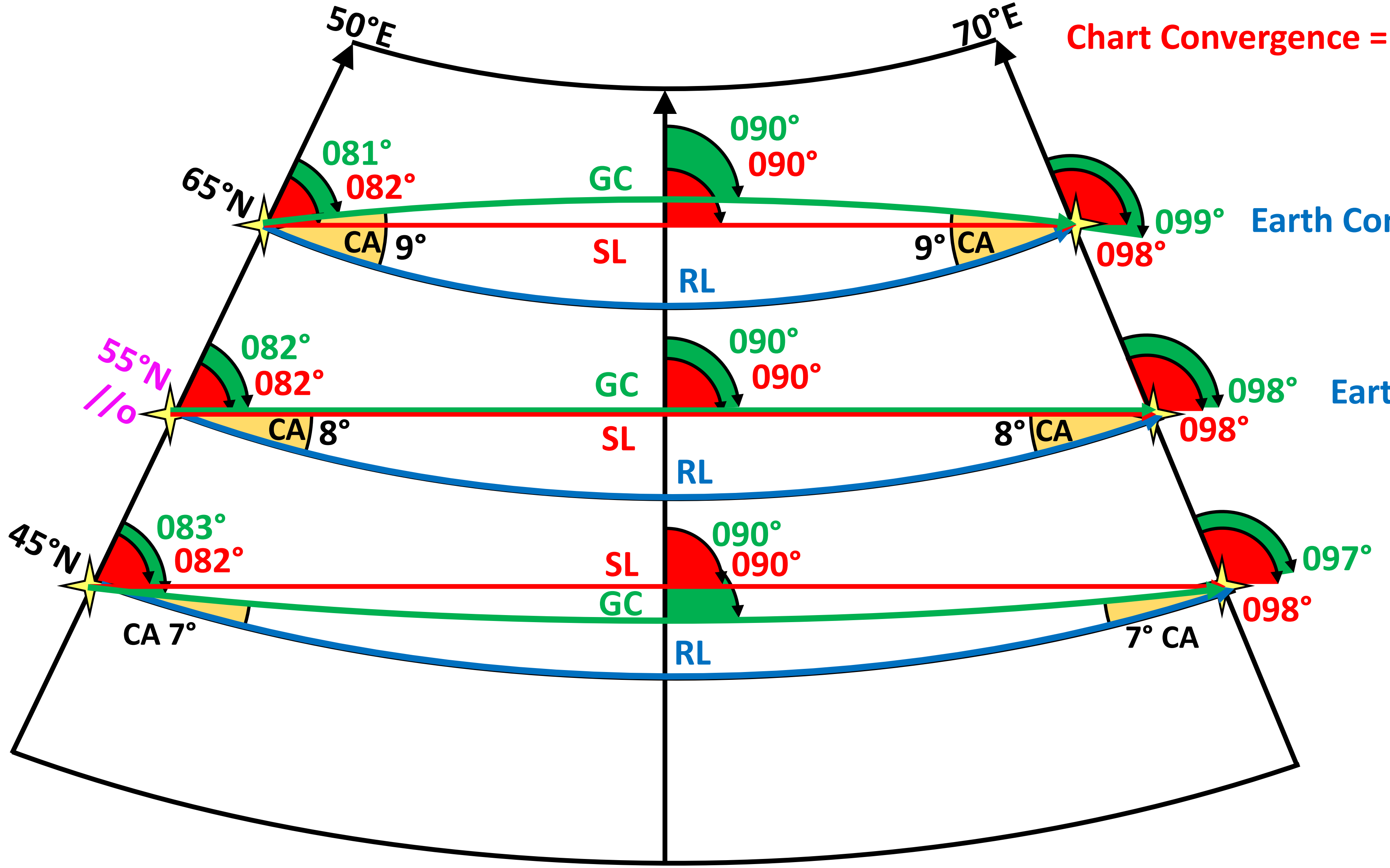
So the **Great Circles** are Straight Lines at the Parallel of Origin and curves concave to the Parallel of Origin at any other latitude.



However, the amount of curvature from the straight line is exaggerated in the Figure above simply to make the explanation clearer. In fact there is very little curvature compared with a Straight Line and, for all practical purposes, including plotting, **Great Circles on a Lambert chart may be treated as Straight Lines**. The demonstration is in the next page.

Chart Convergence = ChLONG x sin//o = 20° x sin55° ≈ 16°

So Conversion Angle on chart = 8°



Earth Convergence = 20° x sin65° ≈ 18°

So Conversion Angle = 9°

Earth Convergence = 20° x sin55° ≈ 16°

So Conversion Angle = 8°

Earth Convergence = 20° x sin45° ≈ 14°

So Conversion Angle = 7°

Track	Initial	Halfway	Final
RL	090°	090°	090°
Map/SL	082°	090°	098°
GC 55°N	082°	090°	098°
GC 45°N	083°	090°	097°
GC 65°N	081°	090°	099°

We see that, except at the Parallel of Origin where the Straight Line is actually a Great Circle, the Great Circle is always **concave** to the Parallel of Origin. On the diagram the curves are exaggerated in order to be seen, however the difference between a Straight Line and a Great Circle is very small. So we can assume that on a Lambert Projection, a Straight Line is Great Circle, as long as plan a navigation near the Parallel of Origin.

SUMMARY OF LAMBERT PROPERTIES

A summary of Lambert properties is set out in the table. All of these are asked in examinations and they should be learnt.

PROPERTIES OF A LAMBERT CHART	
Scale	Correct on the standard parallels. Contracted within the standard parallels. (least at parallel of origin). Expanded outside standard parallels.
Orthomorphic	Yes. All charts used for navigation must be.
Graticule	Meridians are straight lines, originating from the pole. Parallels are arcs of circles, centred at the pole. (The pole is always off the map).
Parallel of Origin	Mathematical basis of projection. Assumed to be halfway between the 2 standard parallels.
Shapes	Reasonably correct over small areas. Distortion over large areas, especially at high latitudes.
Chart Convergence	Constant across the chart. **Chart convergence = ChLONG x sin Parallel of Origin**
Rhumb lines	Meridians are straight lines. All other Rhumb Lines are concave to the pole (ie, parallels of latitude).
Great Circles	Meridians are straight lines. At the parallel of origin - near-straight line. At any other latitude, a curve concave to the parallel of origin.

Exercise

a. The Standard Parallels of a Lambert's conical orthomorphic projection are 07°40'N and 38°20' N. The constant of the cone for this chart is:

b. The constant of cone of a Lambert conformal conic chart is quoted as 0.6455. At what latitude on the chart is Earth convergency correctly represented?

c. A Lambert conformal conic chart, whose standard parallels 50°N and 65°N is used for navigation. Straight Line from A (58°N, 165°E) to B (62°N, 145°E) is drawn on the chart. The true course of the straight line track drawn on this chart at A is 301°. The true course of the straight line track drawn on this chart at B is:

d. A straight line from C (49°S, 155°E) to D (49°S, 170°W) is drawn on a Lambert Conformal conical chart with Standard Parallels at 47°S and 59°S. When passing 175°W, the True Track is:

The correction is in the next pages…

Exercise *(corrected)*

a. The Standard Parallels of a Lambert's conical orthomorphic projection are 07°40'N and 38°20' N. The constant of the cone for this chart is:

Constant of the cone ‘n’ = sin Parallel of Origin

- Parallel of Origin = (7°40’ + 38°20’)/2 = 23°

Constant of the cone ‘n’ = sin 23°

Constant of the cone ‘n’ = 0.39

b. The constant of cone of a Lambert conformal conic chart is quoted as 0.6455. At what latitude on the chart is Earth convergency correctly represented?

The Earth convergency is correctly represented at the Parallel of Origin.

Constant of the cone ‘n’ = sin Parallel of Origin

$\leftrightarrow$ Parallel of Origin = $\sin^{-1}$(Constant of the cone ‘n’)

Parallel of Origin = $\sin^{-1}(0.6455)$

Parallel of Origin ≈ 40.2° = 40°12’

Exercise *(corrected)*:

c. A Lambert conformal conic chart, whose standard parallels 50°N and 65°N is used for navigation. Straight Line from A (63°N, 165°E) to B (65°N, 145°E) is drawn on the chart. The true course of the straight line track drawn on this chart at A is 301°. The true course of the straight line track drawn on this chart at B is:

- On Lambert chart, we can assume that a Straight Line is a Great Circle
- The route between two points on a chart changes by the chart convergence of the meridians at these two points.

Chart Convergence [A/B] = ChLONG [A/B] x sin Parallel of Origin

- Parallel of Origin = (50° + 65°)/2 = 57.5°

Chart Convergence [A/B] = 20° x sin 57.5°

Chart Convergence [A/B] ≈ 17°

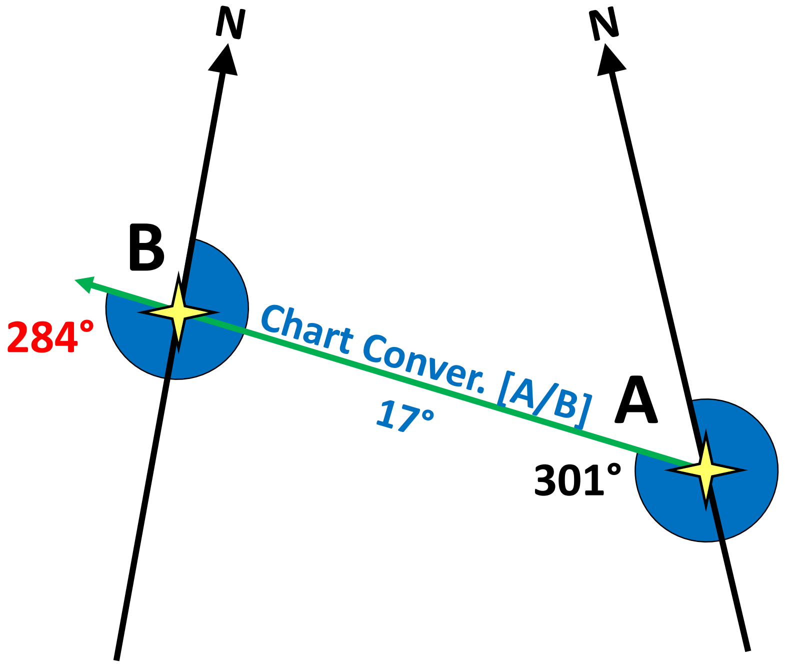
Now we know that the route on the chart between these two points will change by 17°:

Sso now let's see how the Great Circle track increases and decreases:

D	I
I	D

Since the location of the point B is West of the point A, and we are in the Northern Hemisphere, so the route at B decreases compared to the route at A, and it decreases by the chart convergence between A and B.

Therefore, the route on the chart at B is 284°



Exercise *(corrected)*:

d. A straight line from C (49°S, 155°E) to D (49°S, 170°W) is drawn on a Lambert Conformal conical chart with Standard Parallels at 47°S and 59°S. When passing 175°W, the True Track is:

- On Lambert chart, we can assume that a Straight Line is a Great Circle
- C and D are on the same latitude, it means that the Rhumb Line between C and D is their own parallel of latitude that cuts all meridians at 90°. So the the Rhumb Line Track between C and D is 090° or 270°
- The route between C and D is easterly, so the Rhumb Line Track between C and D is 090°
- If the Rhumb Line Track between C and D is 090°, so the Great Circle Track halfway between C and D is 090°
- The **meridian** located halfway between C and D is **172.5°E**, **so at 172.5°E the Great Circle Track is 090°**

So now that we have a Great Circle Track in one location, we can calculate any Great Circle Track at any location between C and D.

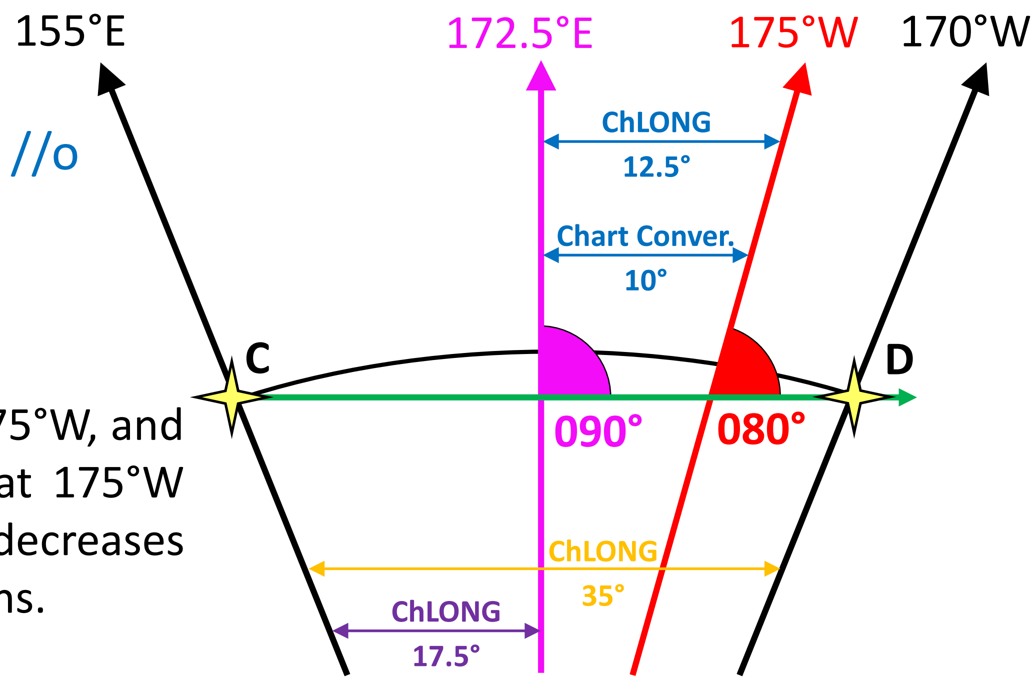
The route between two points on a chart changes by the chart convergence of the meridians at these two points.

Chart Convergence [172.5°E/175°W] = ChLONG [172.5°E/175°W] x sin //o

- Parallel of Origin = (47° + 59°)/2 = 53°

Chart Convergence [172.5°E/175°W] = 12.5° x sin 53°

Chart Convergence [172.5°E/175°W] ≈ 10°



D	I
I	D

Since the meridian 172.5°E is West of the meridian 175°W, and we are in the Southern Hemisphere, so the route at 175°W decreases compared to the route at 172.5°E, and it decreases by the chart convergence between these two meridians.

172.5°E 090° → **Chart Convergency** 10° → **175°W** 080°

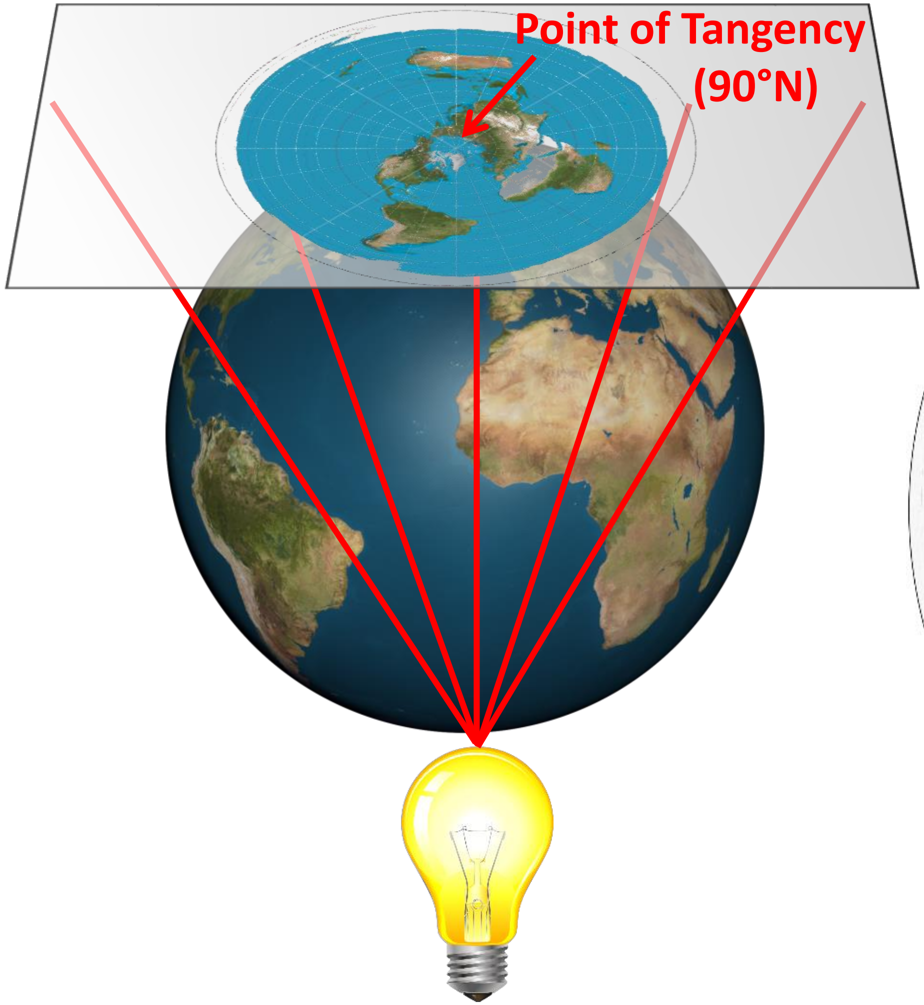
Therefore, the route on the chart at 175°W is 080°

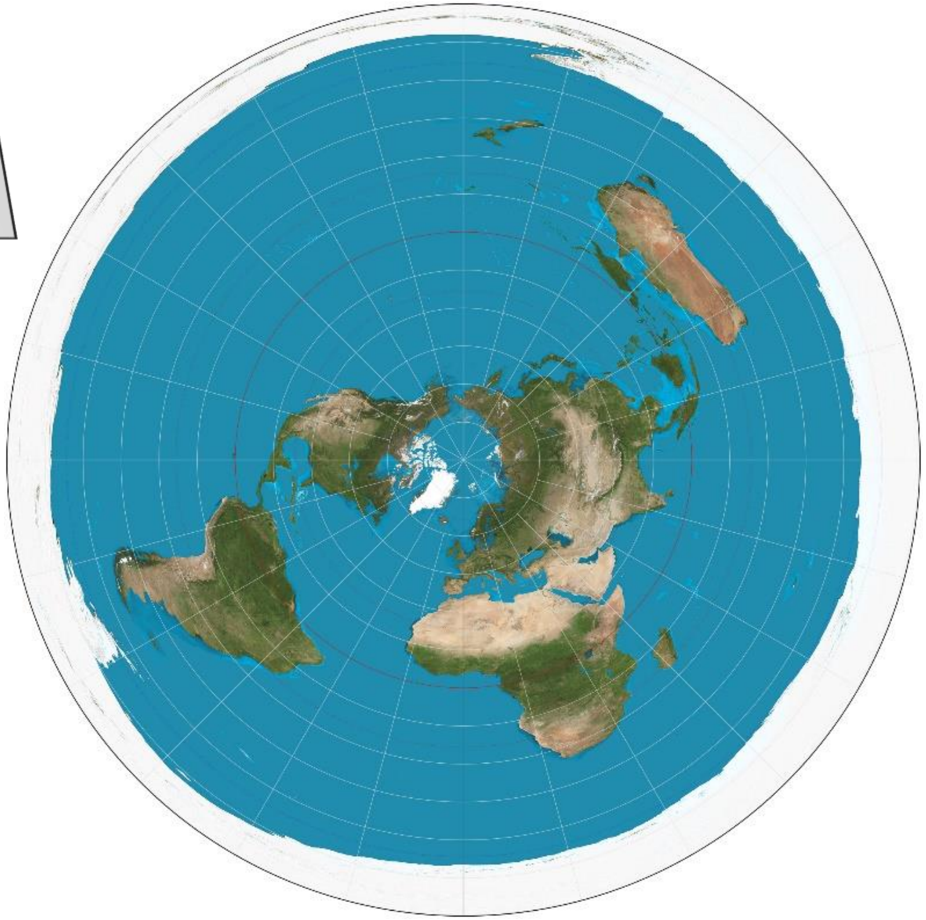
Polar Projection

Polar Stereographic projection

While it may have been used by ancient Egyptians for star maps in some holy books, the earliest text describing the azimuthal equidistant projection is an 11th-century work by the Persian mathematician and astronomer Abu Rayhan Al-Biruni (973-1048).

Abu Rayhan Al-Biruni (973–1048)





The **polar stereographic projection** or **plan projection** is, of those discussed, the only geometric projection, i.e. true **perspective** projection. It is constructed using the principle shown in the Figure avobe

A flat surface is used, touching the North Pole (the point of tangency). The light source is positioned at the South Pole (diametrically opposed), creating a graticule, by geometrical projection, which is shown in the lower part of the diagram.

Scale

The scale is correct at the Pole (where the paper touches the Reduced Earth).
Elsewhere it expands as the **sec² (½ co-latitude)**.
The equation to find scale at any latitude is as follows: **Scale at any latitude = scale at Pole × sec² (½ co-latitude)**

Let's see how this works with a practical example. Take a case where the scale at the Pole is 1/1,000,000. Now let's calculate what the scale on a Polar chart will be at 78°N (or S) latitude.

If the latitude is 78°, then the co-latitude is 12°. Half the co-latitude is 6°. The equation therefore becomes:

$$\text{Scale at } 78^\circ = \frac{1}{1\,000\,000} \times \sec^2(6^\circ) = \frac{1}{1\,000\,000} \times \sec 6^\circ \times \sec 6^\circ = \frac{1}{1\,000\,000} \times \frac{1}{\cos 6^\circ} \times \frac{1}{\cos 6^\circ}$$

There is no point to do the entire calculation of each fraction. The result expected to be found is $\frac{1}{number}$. So simply multiply the denominators and the result put is as the denominator in the scale of that latitude:

$$\text{Scale at } 78^\circ = \frac{1}{\boxed{1\,000\,000}} \times \frac{1}{\boxed{\cos 6^\circ}} \times \frac{1}{\boxed{\cos 6^\circ}} = \frac{1}{\boxed{\text{Distance at } 78^\circ}} = \frac{1}{989\,074}$$

$\times \quad \times \quad =$

In other words, between latitudes 90° and 78°, the scale is within 1% of the scale at the pole, which can be regarded as a constant scale chart.
A similar exercise can be carried out for the latitude of 70°. Out to 70° the scale is within 3% of the scale at the Pole.
To summarise:
Between latitudes 90° and 78° - scale within 1% of scale at Pole.
Between latitudes 78° to 70° - scale between 1% and 3% of scale at Pole.

Orthomorphism

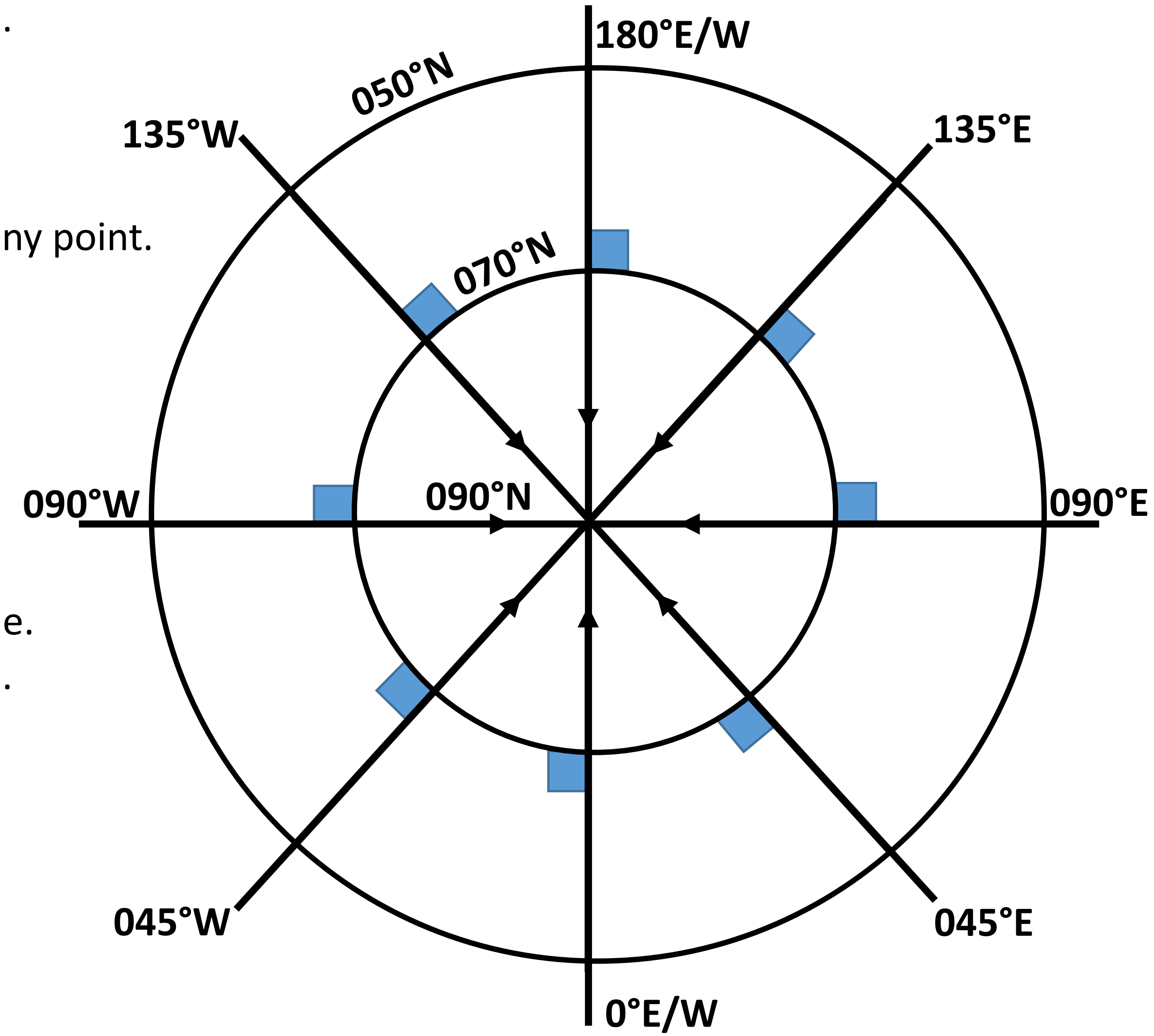
Meridians are straight lines originating from the pole.

Parallels of latitude are arcs of circles centred at the pole.

The meridians and the parallels intersect at 90° angle.

The expansion is at the same rate in any direction from any point.

Therefore the projection is **orthomorphic**.

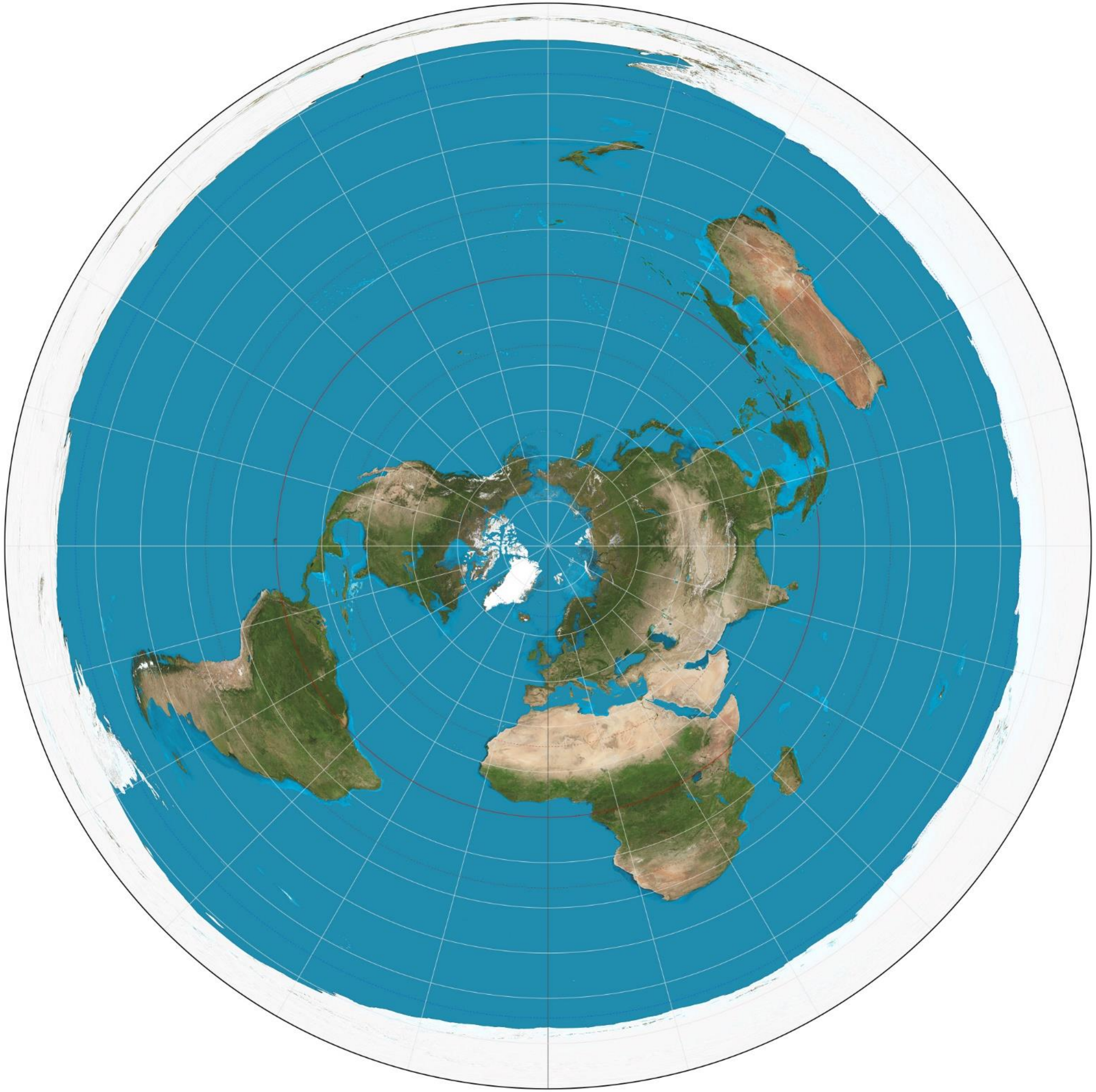


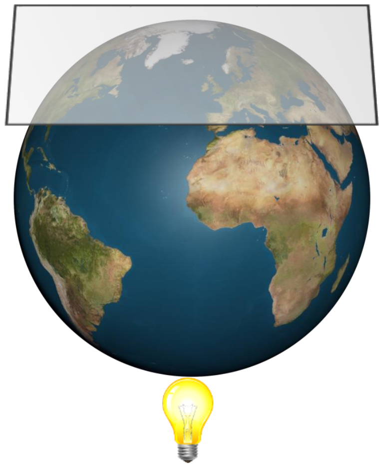
Graticule

Meridians are straight lines originating from the pole. Parallels of latitude are arcs of circles centred at the pole.

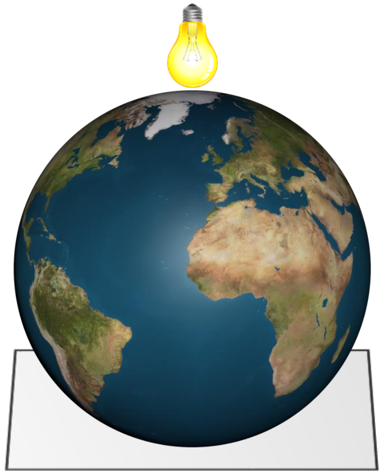
Shapes

Because of scale expansion, shapes and areas will be distorted away from the pole.





Be aware of the map shape for the Polar Projection in the Southern Hemisphere.
For the Northern Hemisphere, the North is in the center of the map.
For the Southern Hemisphere, the North is on the edge around of the map.
Notice: At the first step when exactly at the North Pole, you can only proceed South, and when exactly at the South Pole, you can only proceed North.



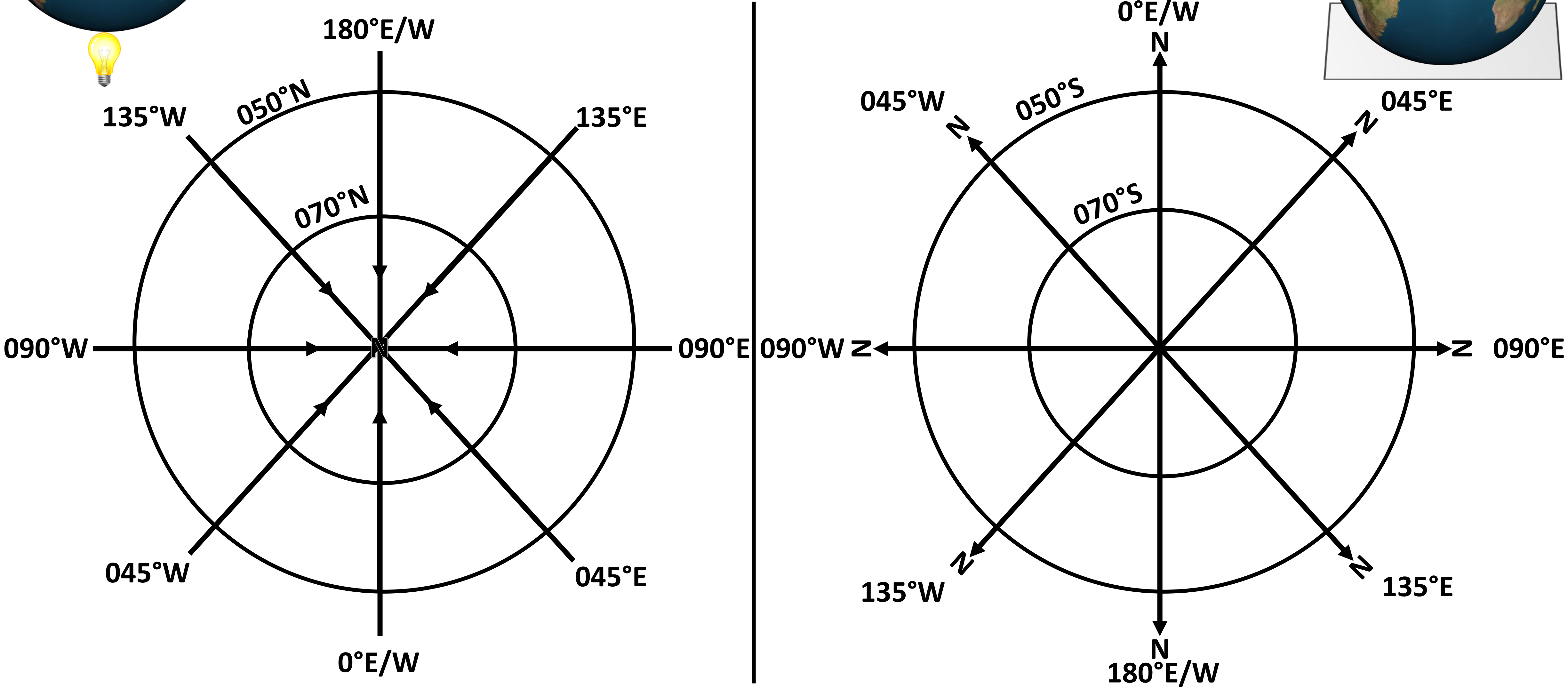


Chart Convergence

If we take we compare the most the green meridians of the chart at a given latitude (ie, 50°N), we can see that their angle of convergency on the chart is actually the change of longitudes in between.

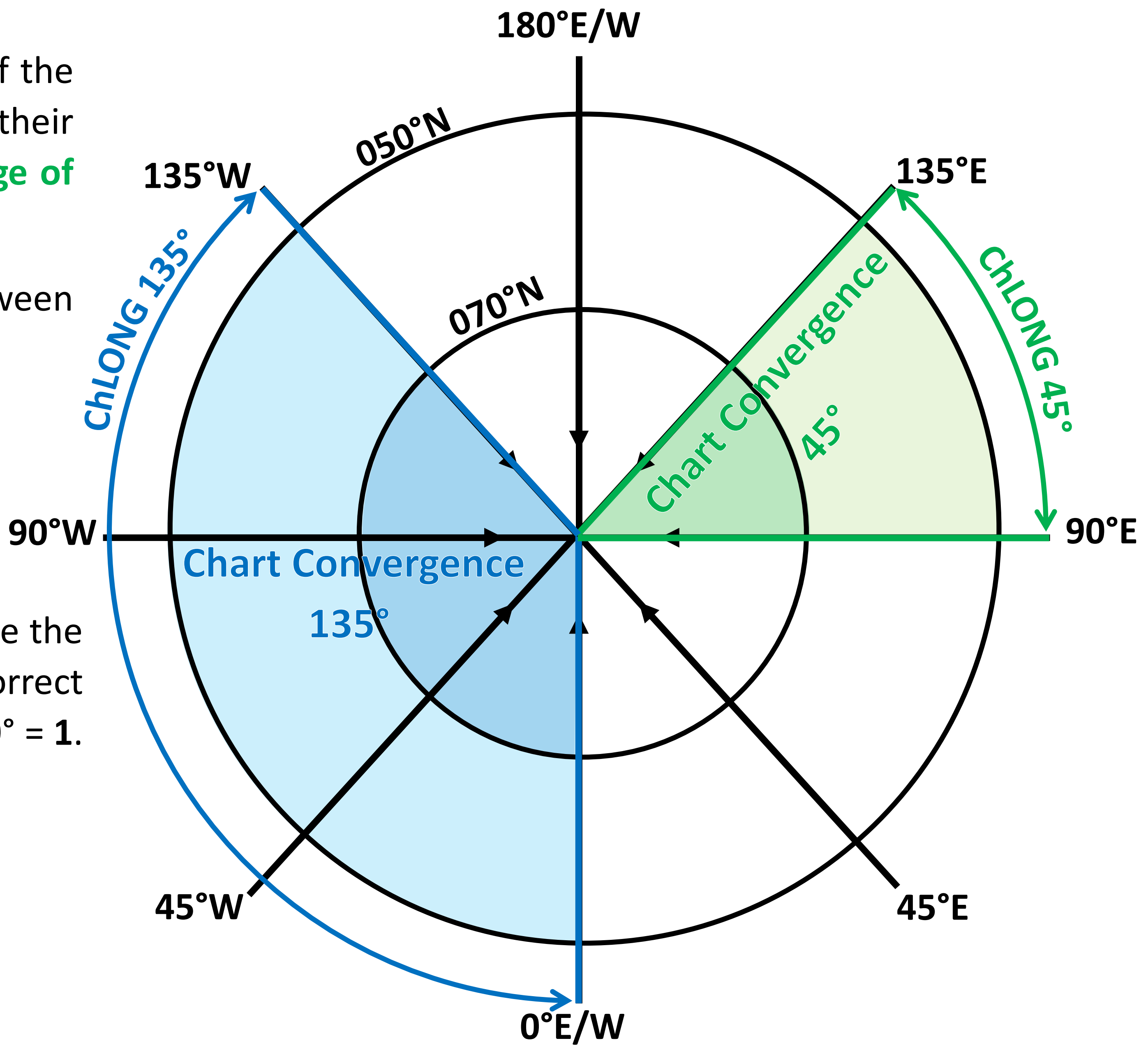
However we notice that the chart convergency between these meridians is the same at any latitude (ie, 70°N)

If we repeat the same verification with the blue meridians, we can see that the chart convergency in between is their change of longitudes, and at any latitude.

The chart convergence is constant across the chart. Since the chart is projected the pole, the chart convergence is correct at the Pole only, with the **convergency factor 'n'** = sin 90° = **1**.

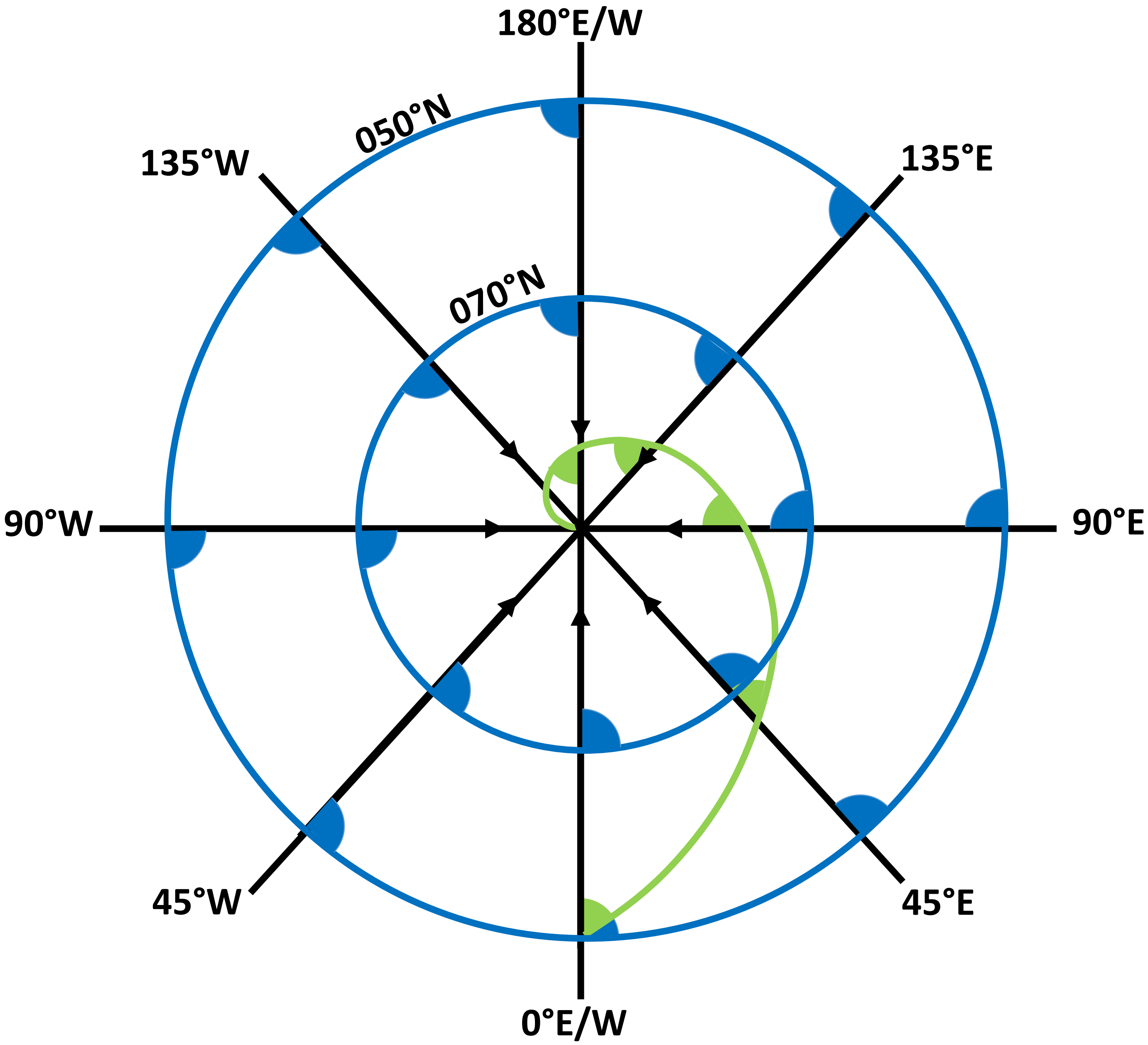
So on Polar Projection, the chart convergency depends only on the change of longitudes.

Chart Convergence = ChLONG



Rhumb Lines

Except for meridians which appear as straight lines, Rhumb Lines are curves concave to the pole of the projection.



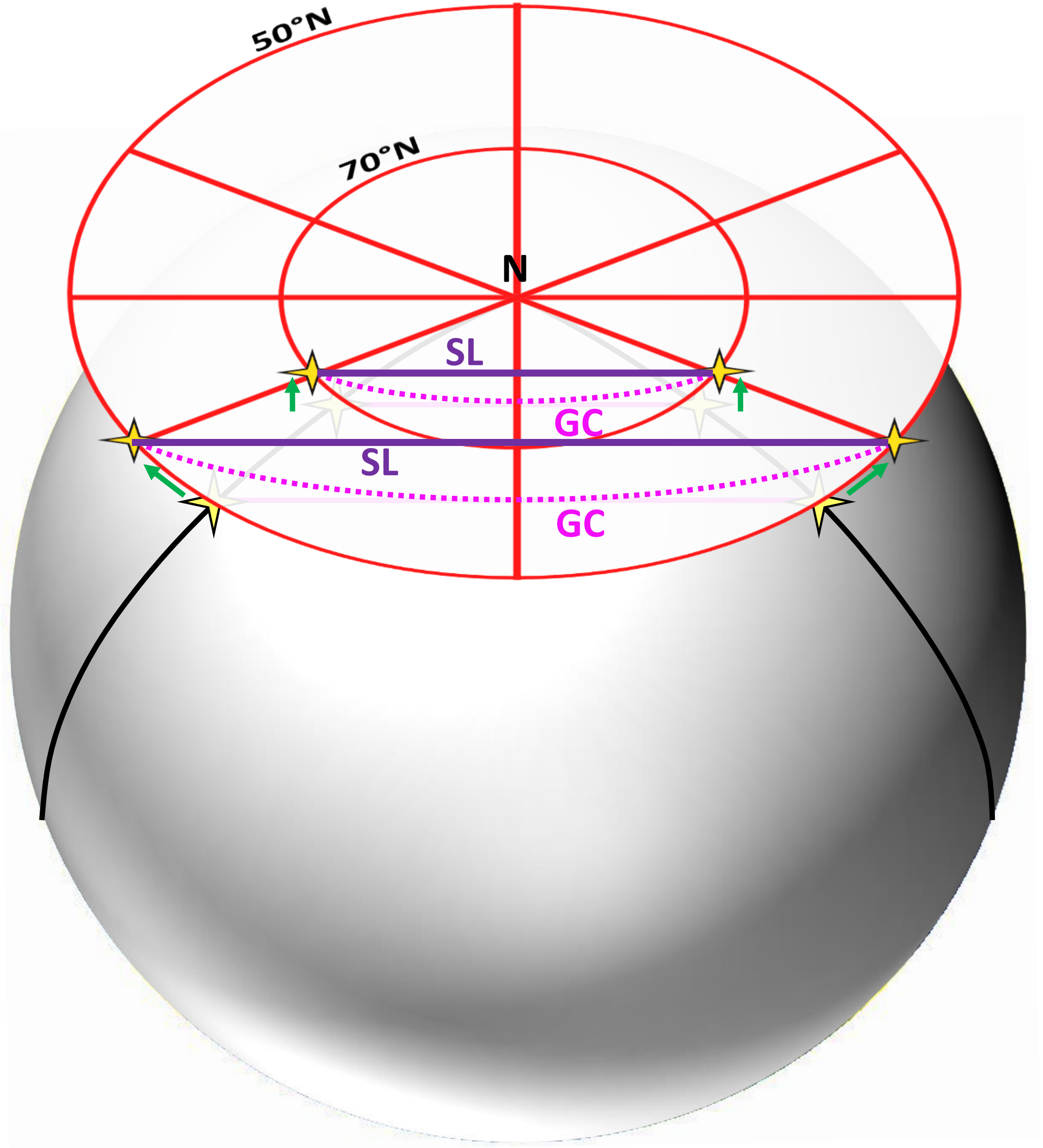
Great Circles

Except for the meridians which appear as Straight Lines, Great Circles appear as curves concave to the Pole.

When planning a navigation. A **Straight Line** is drawn between two points.

At high latitudes, a the **Great Circle** approximated the **Straight Line**.

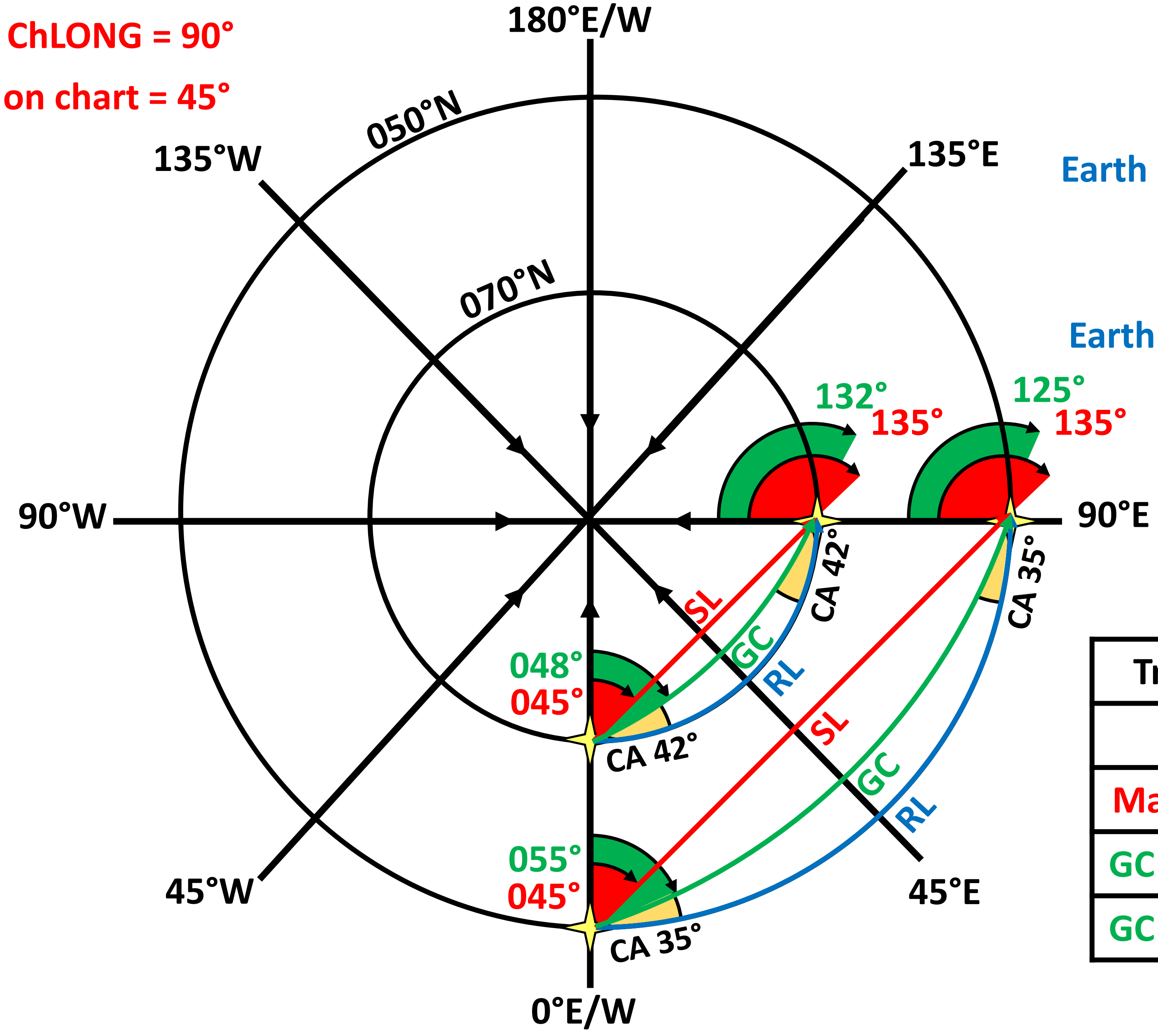
Where a lower latitudes, there will be a big difference between a **Straight Line** and a **Great Circle**.



So we can consider that on a Polar chart, at high latitudes (from 70°N/S and higher), that a **Straight Line** is a **Great Circle**. As demonstrated in the next slides.

Chart Convergence = ChLONG = 90°

So Conversion Angle on chart = 45°



Earth Convergence = 90° x sin50° ≈ 70°

So Conversion Angle = 35°

Earth Convergence = 90° x sin70° ≈ 84°

So Conversion Angle = 42°

Track	Initial	Halfway	Final
RL	090°	090°	090°
Map/SL	045°	090°	135°
GC 50°N	055°	090°	125°
GC 70°N	048°	090°	132°

From 70°N/S and higher, there is a very small difference between a **Straight Line** and a **Great Circle**. As demonstrated in the next slides. So we can consider that a **Straight Line** is a **Great Circle**, as long as this projection is used for latitude 70°N/S or higher.

SUMMARY OF POLAR PROPERTIES

A summary of Polar properties is set out in the table. All of these are asked in examinations and they should be learnt.

PROPERTIES OF A POLAR CHART	
Scale	Correct at Pole. Elsewhere expands as $\sec^2$ (½ co- latitude) Within 1% from latitudes 90° to 78°. Within 3% from latitudes 78° to 70°.
Orthomorphic	Yes. All charts used for navigation must be.
Graticule	Meridians are straight lines radiating from the Pole. Parallels are concentric circles drawn from the pole.
Shapes	Become more distorted as distance increases from the Pole.
Chart Convergence	Correct at the Pole. Constant across the chart. **Convergence = ChLONG** ‘n’ = 1
Rhumb lines	Curves concave to the nearer Pole.
Great Circles	Curves concave to the nearer Pole, but with less curvature than Rhumb Lines. Can be taken as straight lines at latitudes greater than 70°.

Exercise

a. What is the initial straight line from A (75°N 030°W) to B (75°N 090°E) on Polar Stereographic chart?

b. What is the initial straight line from C (75°S 160°W) to D (75°S 100°E) on Polar Stereographic chart?

c. At what longitude does the straight line track from E (70°N 40°W) to F (70°N 80°E) on a Polar Stereographic chart reach its highest latitude?

d. On a Polar Stereographic map, a line is drawn from position G (70°N 102°W) to position H (80°N 006°E). The point of highest latitude along this line occurs at 035°W. What is the initial straight-line track angle from G to H, measured at G?

The correction is in the next pages…

Exercise *(corrected)*:

a. What is the initial straight line from A (75°N 030°W) to B (75°N 090°E) on Polar Stereographic chart?

- On Polar chart, we can assume that a Straight Line is a Great Circle
- A and B are on the same latitude, it means that the Rhumb Line between A and B is their own parallel of latitude that cuts all meridians at 90°. So the the Rhumb Line Track between A and B is 090° or 270°
- The route between A and B is easterly, so the Rhumb Line Track between A and B is 090°
- If the Rhumb Line Track between A and B is 090°, so the Great Circle Track halfway between A and B is 090°
- From A to halfway, the Great Circle Track will change by ½ of the chart convergence

Chart Convergence [A/B] = ChLONG [A/B]

Chart Convergence [A/B] = 120°

→ From A to halfway, the Great Circle Track will change by ½ of the convergency, so **120°/2 = 60°**

So now let's see how the Great Circle track increases and decreases.

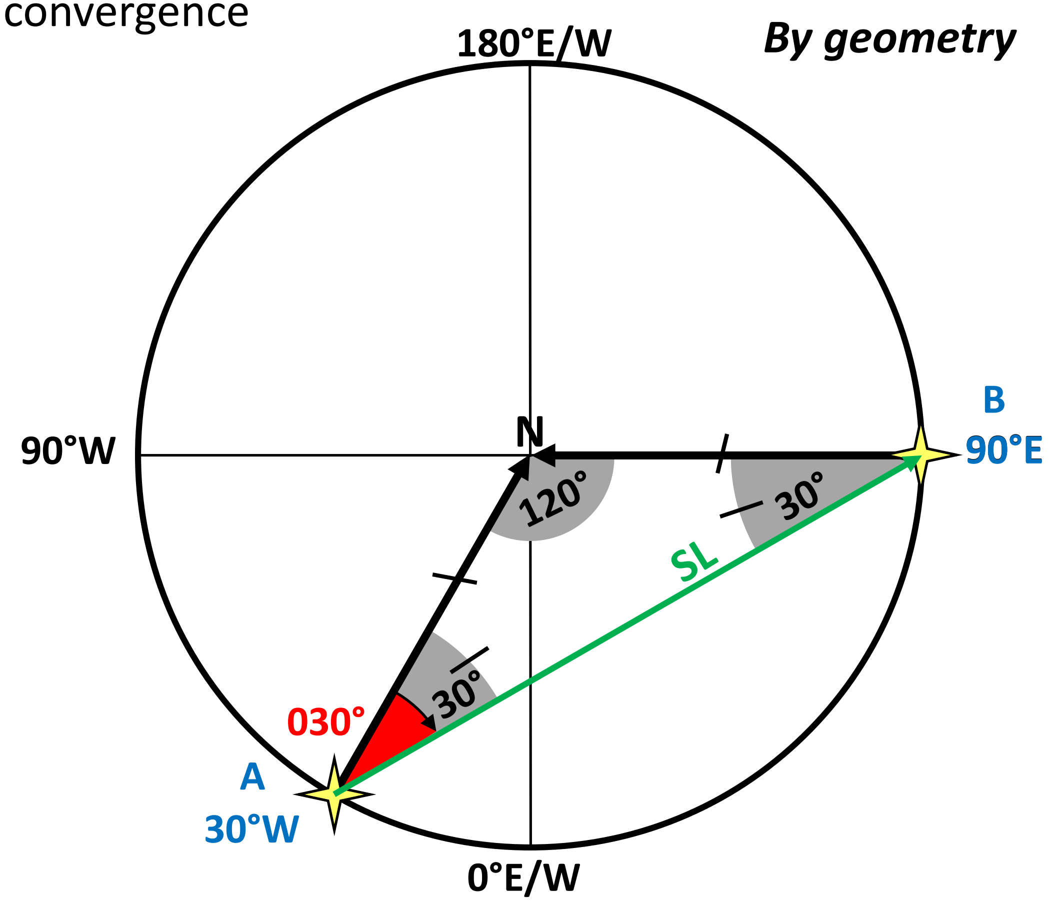
D	I
I	D

Since the location of the point A is West of the point B, so A is also West of the halfway point, and we are in the Northern Hemisphere, so the GCT at halfway point increases compared to the GCT at A, and it will increase by the ½ of the chart convergence between A and B.

A Chart Convergence **½** Chart Convergence **B**

030° 60° **090°** 60°

By geometry



Therefore, the route on the chart at A is 030°

Exercise *(corrected)*:

b. What is the initial straight line from C (75°S 160°W) to D (75°S 100°E) on Polar Stereographic chart?

- On Polar chart, we can assume that a Straight Line is a Great Circle
- C and D are on the same latitude, it means that the Rhumb Line between C and D is their own parallel of latitude that cuts all meridians at 90°. So the the Rhumb Line Track between C and D is 090° or 270°
- The route between C and D is westerly, so the Rhumb Line Track between C and D is 270°
- If the Rhumb Line Track between C and D is 270°, so the Great Circle Track halfway between C and D is 270°
- From C to halfway, the Great Circle Track will change by ½ of the chart convergence

By geometry

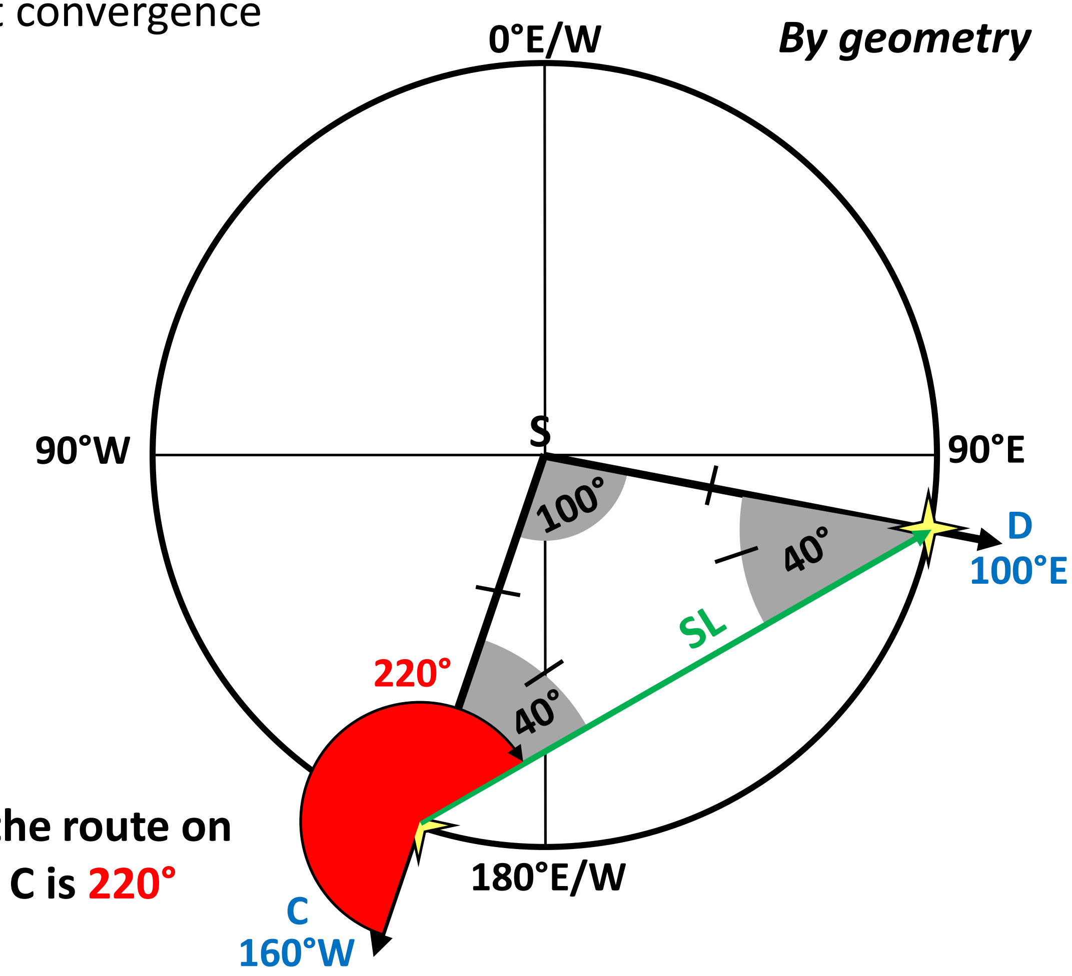


Chart Convergence [C/D] = ChLONG [C/D]

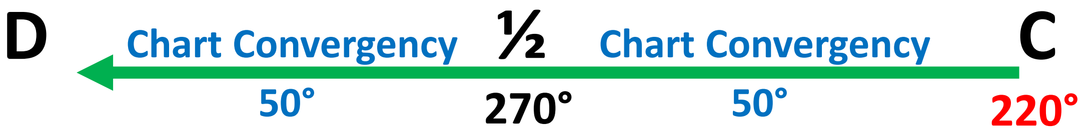
Chart Convergence [C/D] = 100°

→ From A to halfway, the Great Circle Track will change by ½ of the convergency, so **100°/2 = 50°**

So now let's see how the Great Circle track increases and decreases.

D	I
I	D

Since the location of the point C is East of the point D, so C is also East of the halfway point, and we are in the Southern Hemisphere, so the GCT at halfway point decreases compared to the GCT at C, and it will decrease by the ½ of the chart convergence between C and D.



Therefore, the route on the chart at C is 220°

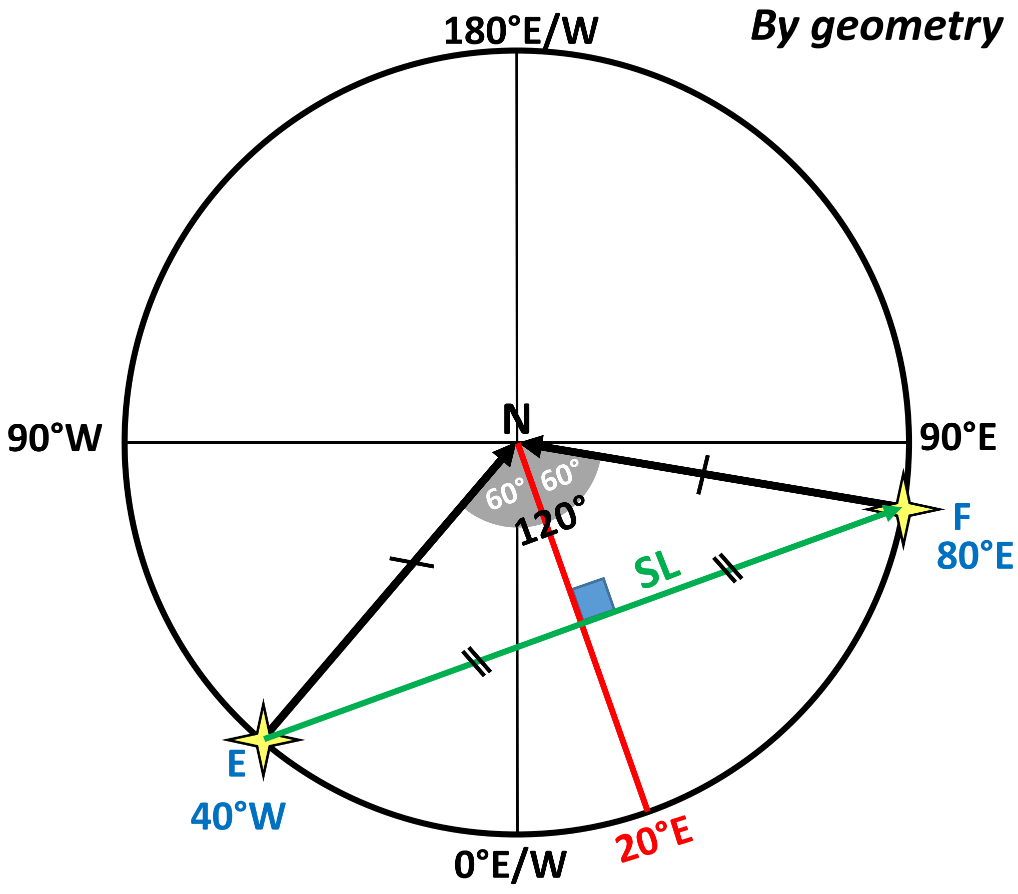
Exercise *(corrected)*:

c. At what longitude does the straight line track from E (70°N 40°W) to F (70°N 80°E) on a Polar Stereographic chart reach its highest latitude?

- On Polar chart, we can assume that a Straight Line is a Great Circle
- Remember that **when passing the highest latitude, the Great Circle Track is 090° or 270°**. So when **two points are on the same latitude**, since the **Great Circle Track halfway** is **090°** or **270°**, the **highest latitude is reached halfway** between these points.

The **meridian** located halfway between E and F is **20°E**.

Therefore, the highest latitude between C and D is reached when passing the longitude 20°E



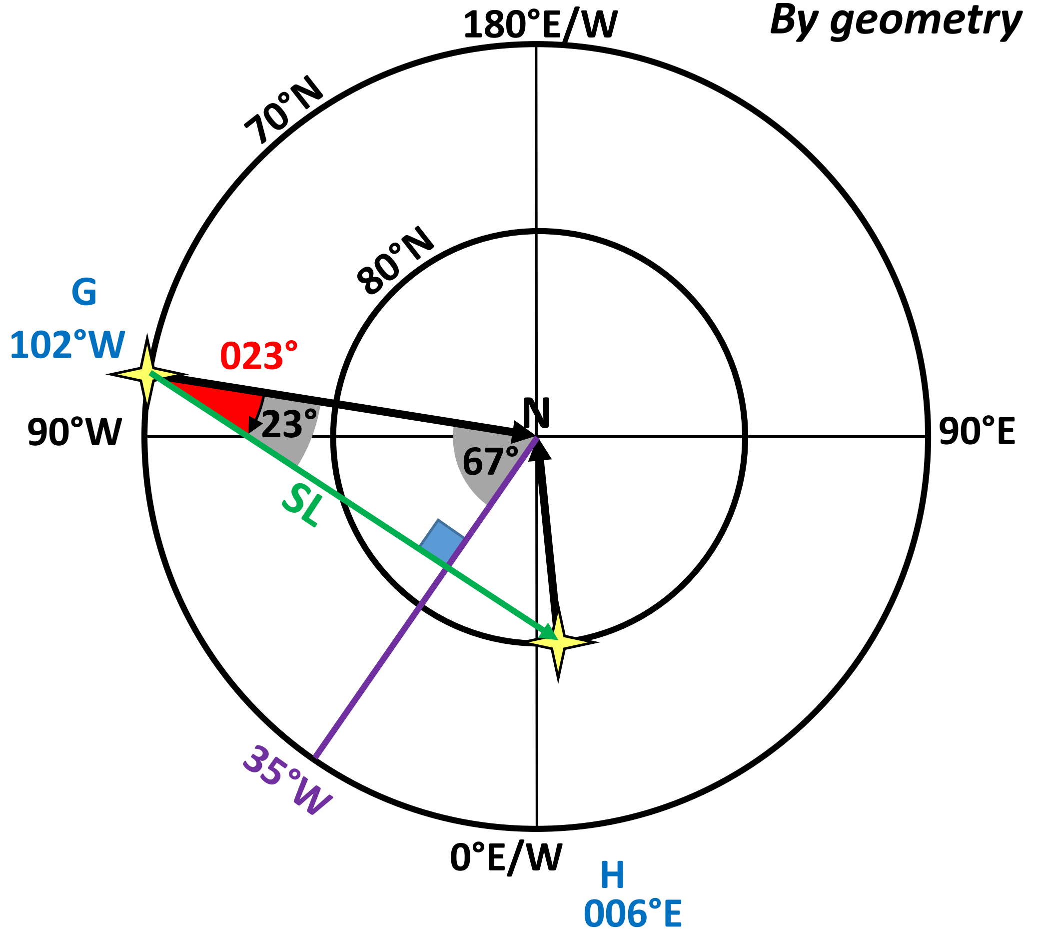
Exercise *(corrected)*:

d. On a Polar Stereographic map, a line is drawn from position G (70°N 102°W) to position H (80°N 006°E). The point of highest latitude along this line occurs at 035°W. What is the initial straight-line track angle from G to H, measured at G?

- On Polar chart, we can assume that a Straight Line is a Great Circle
- Remember that when passing the highest latitude, the Great Circle Track is 090° or 270°
- The route between G and H is easterly so when passing the highest latitude, the Great Circle Track is 090°
- **So the Great Circle track when passing the longitude 035°W is 090°.**

So now that we have a Great Circle Track in one location, we can calculate any Great Circle Track at any location between G and H.

By geometry



The route between two points on a chart changes by the chart convergence of the meridians at these two points.

Chart Convergence [G/035°W] = ChLONG [G/035°W]

Chart Convergence [G/035°W] = 67°

D	I
I	D

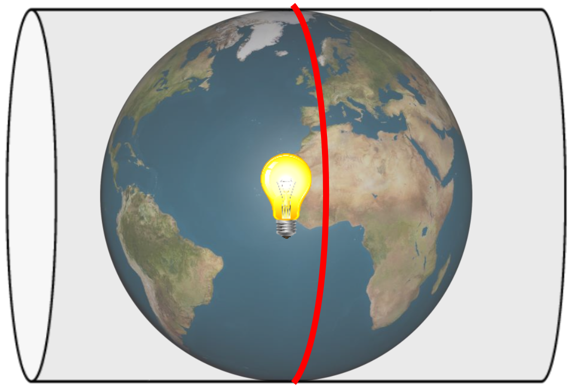
Since the location of the point G is West of the meridian 035°W, and we are in the Northern Hemisphere, so the route at G decreases compared to the route at 035°W, and it decreases by the chart convergence between these two meridians.

$$\underset{023°}{\mathbf{G}} \xrightarrow[\text{67°}]{\text{Chart Convergency}} \underset{090°}{\mathbf{035°W}}$$

Other Projections

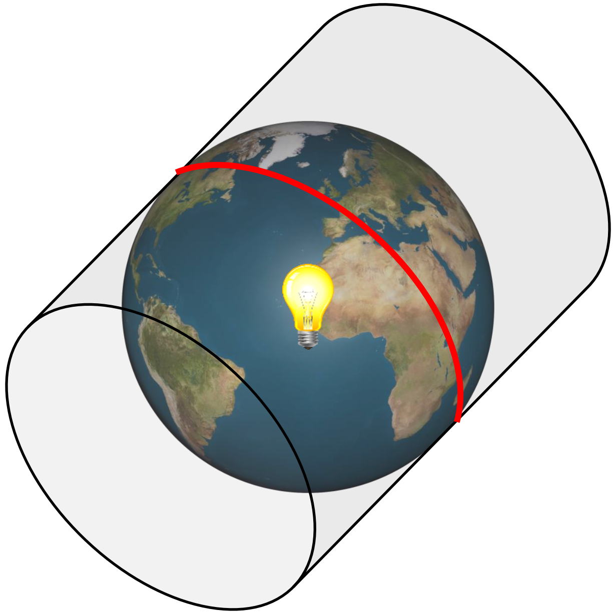
Various projections exist with different property and different purpose of use, and all are subject to study, like:

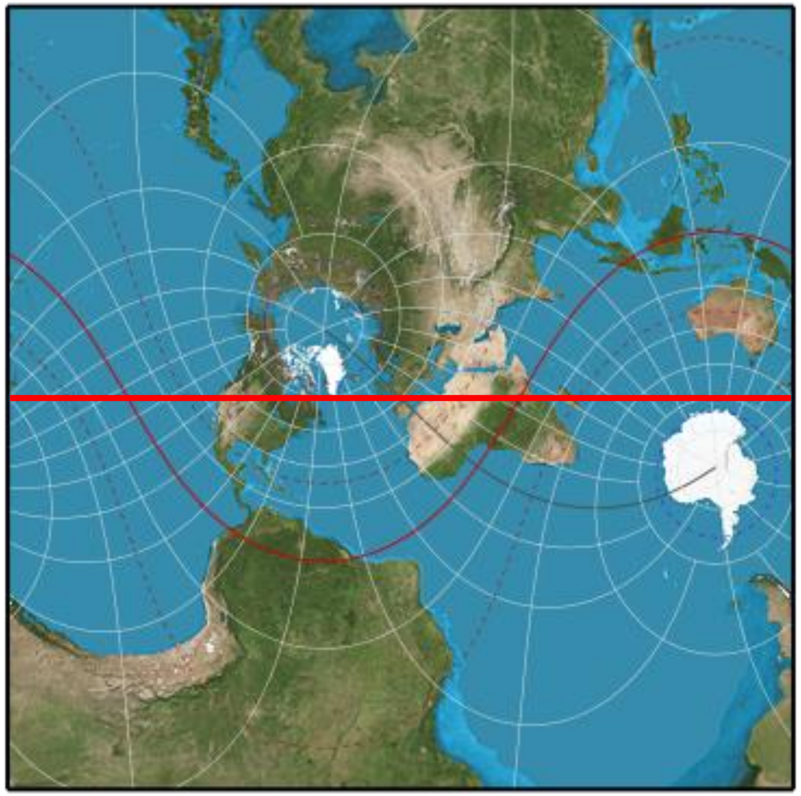
- The **Transverse Mercator**, that maintains the areas corrected over a given meridian and allow accurate navigation North-South along or near that meridian,
- The **Oblique Mercator**, that maintains the areas corrected over a oblique area and allow accurate navigation North-East and South-West or North-West and South-East on that area.
- And others…

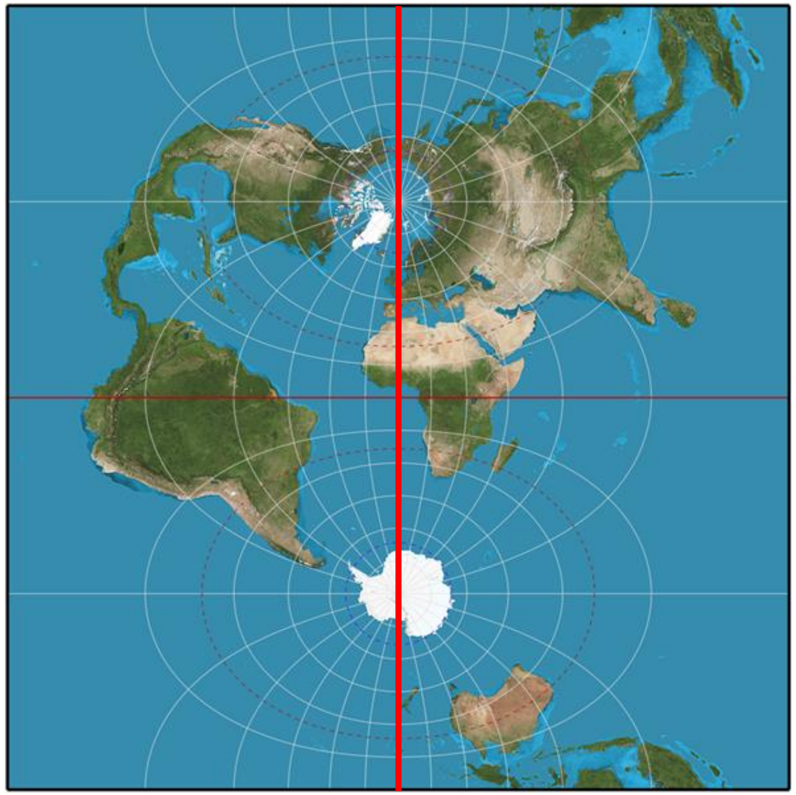


Transverse Mercator

Oblique Mercator

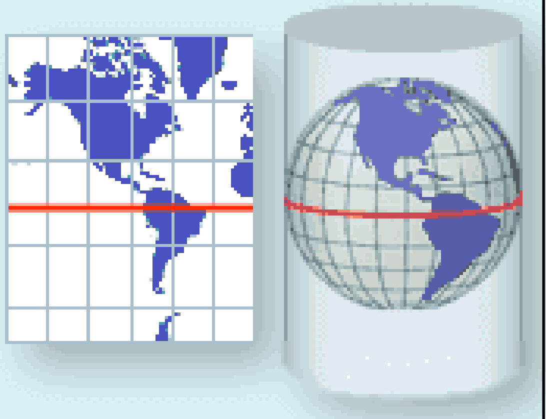
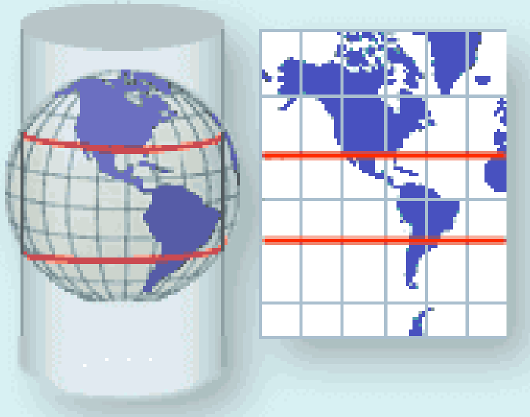
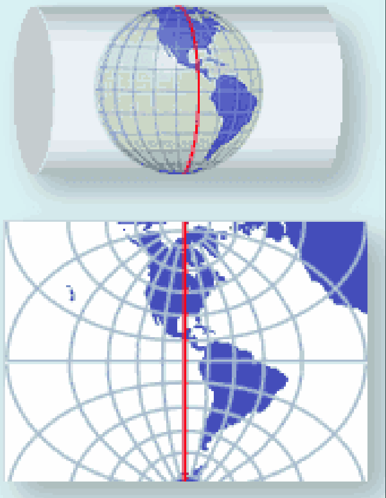
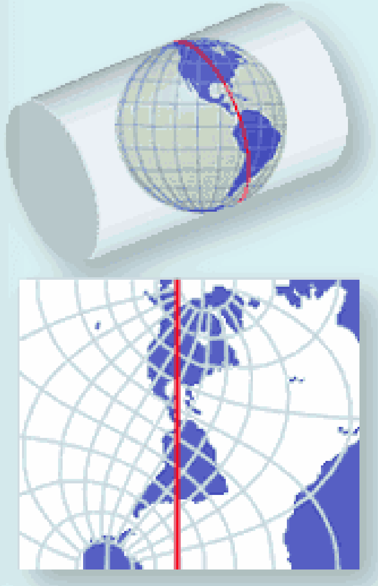


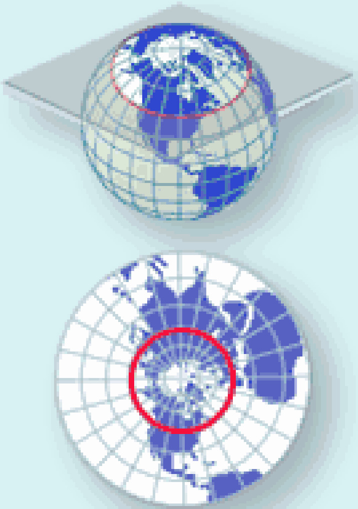
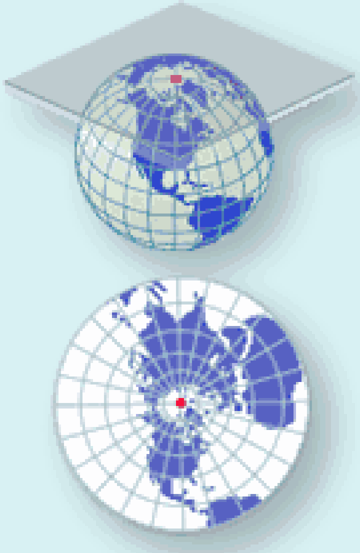


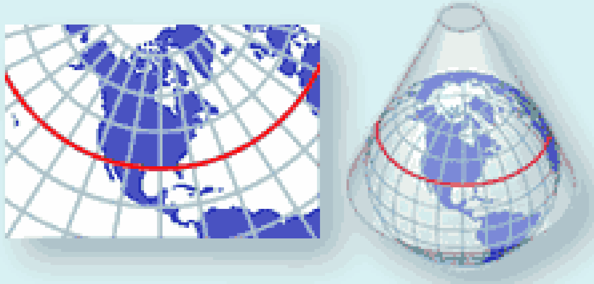
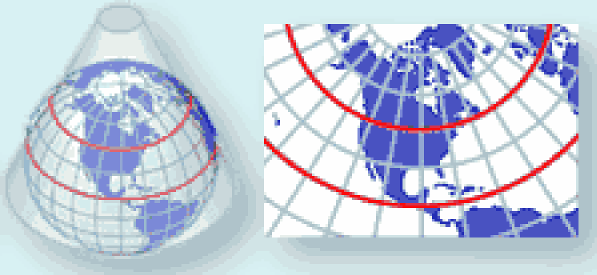


Summary

Any projection can be modified and adjusted in other to satisfy othomorphism or conformality and obtain a better accuracy for navigation (scale, Great Circle, etc.)

Cylindrical			
Direct	**Secant at two latitudes**	**Transverse**	**Oblique**
			

Stereographic or Plan	
Secant	**Tangent**
	

Conical	
Simple	**Orthomorphic**
	

Remember, for the three projections that have been covered (and most of the projections in general),
- the area and the scale will be correct at the parallel of tangency
- the chart convergence is the Earth's convergence at that latitude projected, the same everywhere, and so correct only at that latitude
- The Great Circles always concave to the parallel of tangency, except the meridians which are straight lines and the Great Circles is a straight line at the parallel of tangency

When planning a navigation, we draw a straight line that we will follow, so we choose the map which is projected at the latitudes of the departure and arrival point, or at the mean latitude of these points. So the straight line drawn approximates the Great Circle to follow and the areas navigated are correctly represented on the map, and the navigation is accurate.

Projection	Projected at	Scale	Great Circles	Convergency Factor	Chart Convergence
Mercator	Equator (0°N/S)	Correct at Equator	Concave to the Equator (except the meridians and at the Equator)	0 [sin 0°]	Use Earth Convegrency to determine the real angle Conv = ChLONG x sin Mean Lat
Lambert	Parallel of Origin	Correct at Standard Parallels	Concave to the Parallel of Origin (except the meridians and at the Parallel of Origin)	Constant of the cone 'n' = sin Parallel of Origin	Chart Conv = ChLONG x sin Parallel of Origin
Polar	Pole (90°N/S)	Correct at the Pole	Concave to the nearer Pole (except the meridians and at the Pole)	1 [sin 90°]	Chart Conv = ChLONG