II

CONVERGENCY & CONVERSION ANGLE

We have seen that the Great Circle track changes the value of the angle at each meridian.

In this chapter, we will see how and by how much the Great Circle track changes between two meridians on Earth.

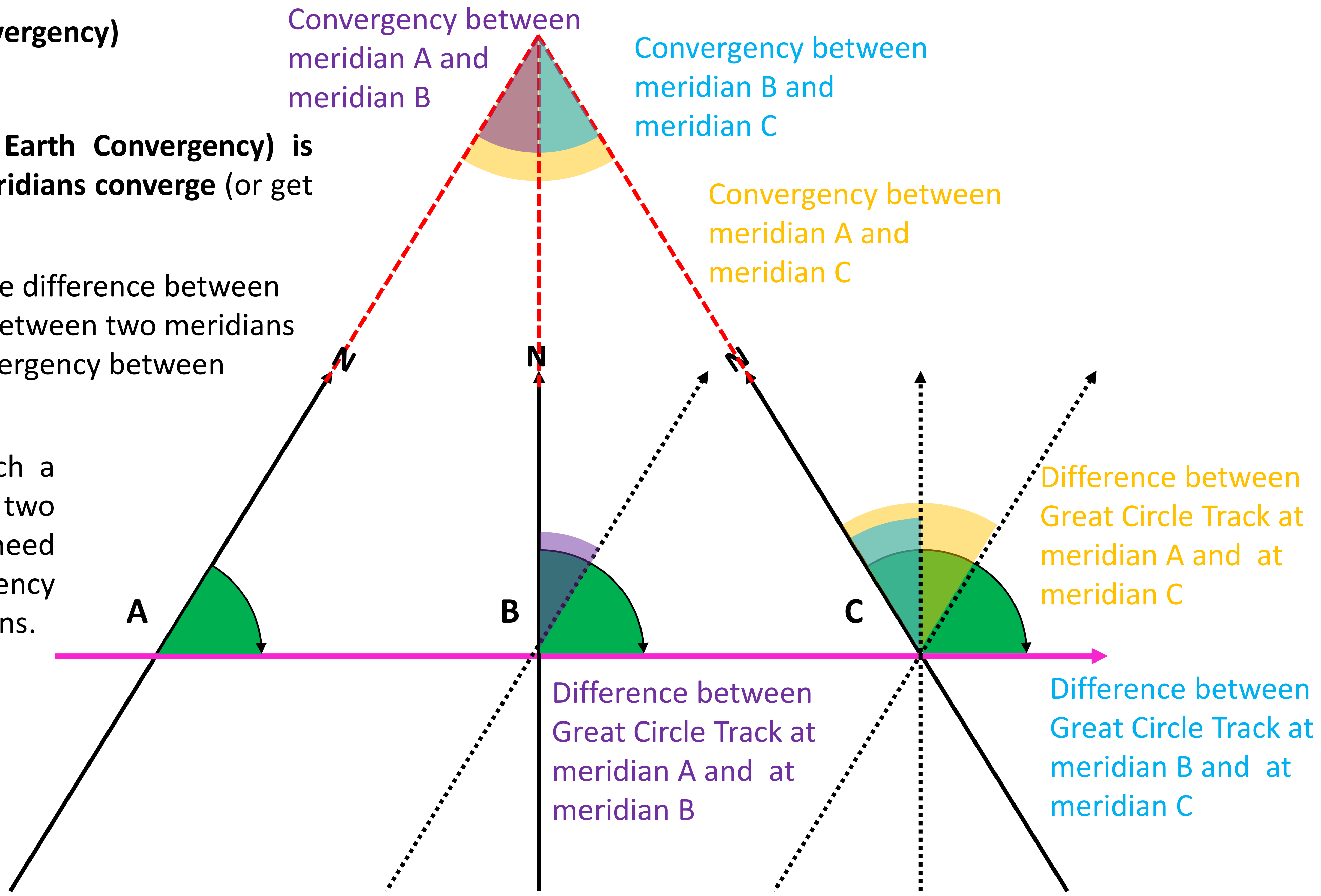
Later when the map projections will be studied in details, we will compare, the route draw by a straight line between two points, with the real Great Circle between those two point. on the real Earth. This is to see if following the straight line draw on the map will be making us flying the Great Circle on Earth or not.

Convergency (or Earth Convergency)

The convergency (or the Earth Convergency) is the angle at which two meridians converge (or get closer in other words).

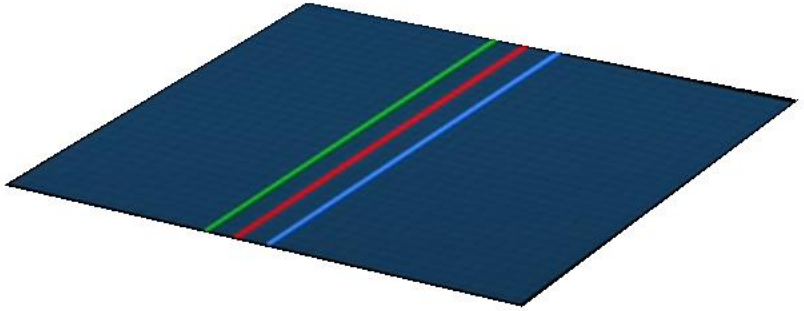
The angular difference or the difference between the Great Circle (GC) track between two meridians is actually equal to the convergency between these two meridians.

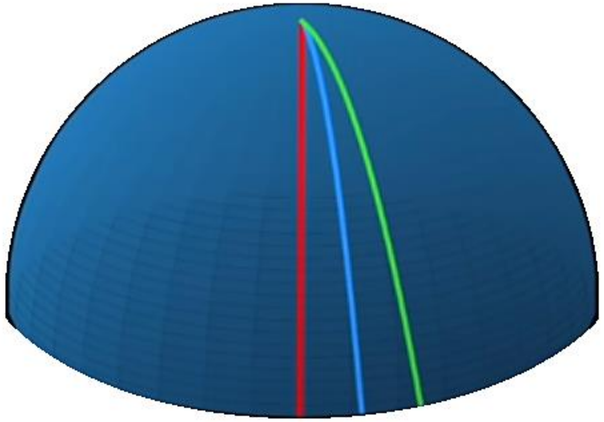
To determine by how much a route will change between two meridians, we therefore need to calculate the convergency between these two meridians.

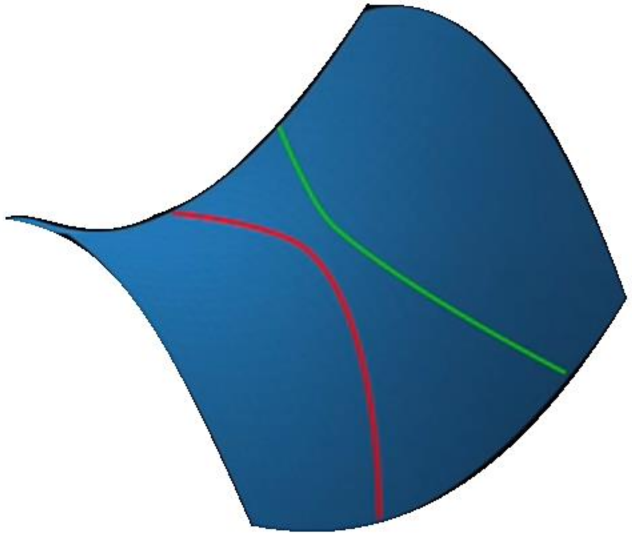


Let’s see how the meridians converge on Earth.

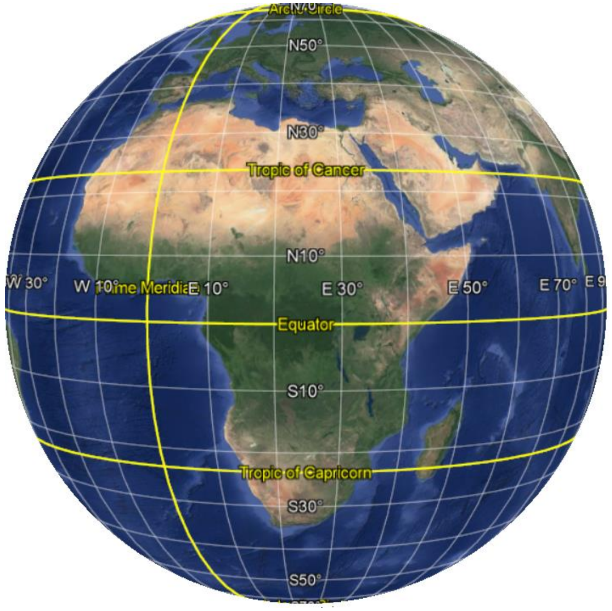
The meridians are parallels lines. On a 2D plane, parallel lines have the same trajectory, however on a curved surface, the parallel line ‘converge’.

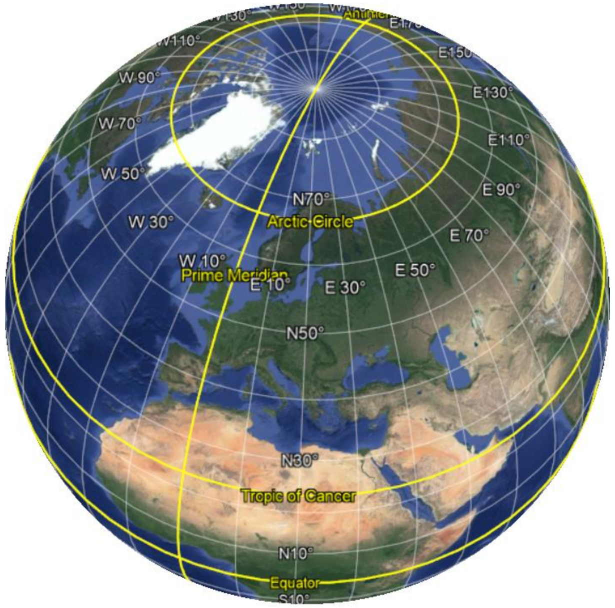


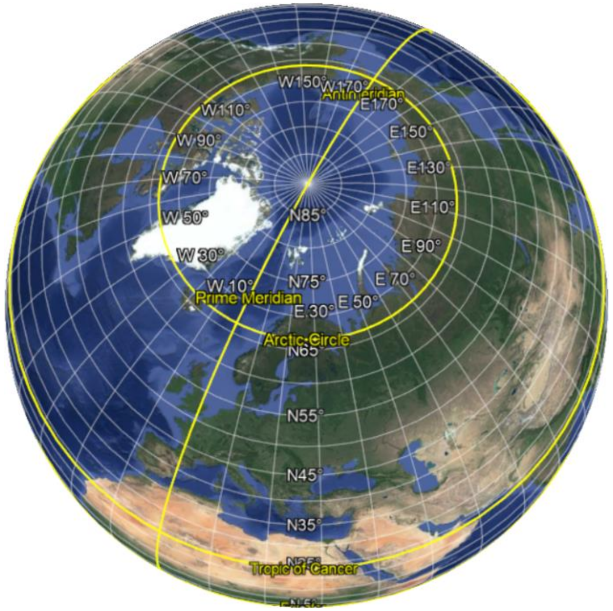




Also, we can see that the meridians don’t converge in the same way based on the latitude at which there a observed







When we observe the meridians from the pole, we can see that they converge by their angular separation, in other terms, the change of longitudes in between.

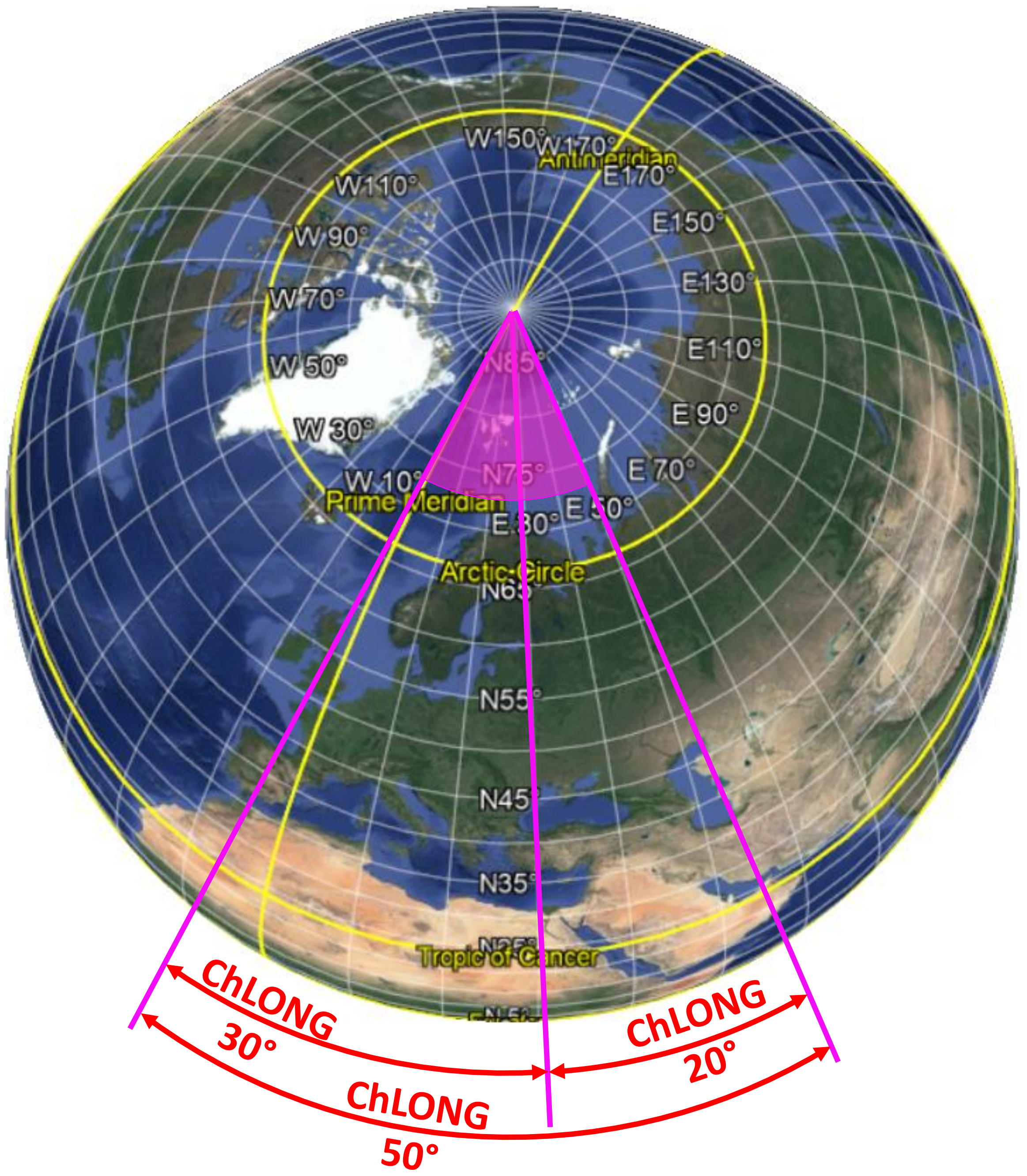
For example, the Prime Meridian (0°E/W) and the meridian 50°E, are separated by 50° and they converge by 50°.

The same, the Prime Meridian (0°E/W) and the meridian 30°E, are separated by 30° and they converge by 30°.

As well, the meridian 30°E and the meridian 50°E, are separated by 20° and they converge by 20°.

So we can say that the convergency at the poles depends on the change of longitude.

Convergency = ChLONG



When we observe the meridians at mid-latitudes (45°N), we can see that they converge as well, but less than their change of longitudes in between.

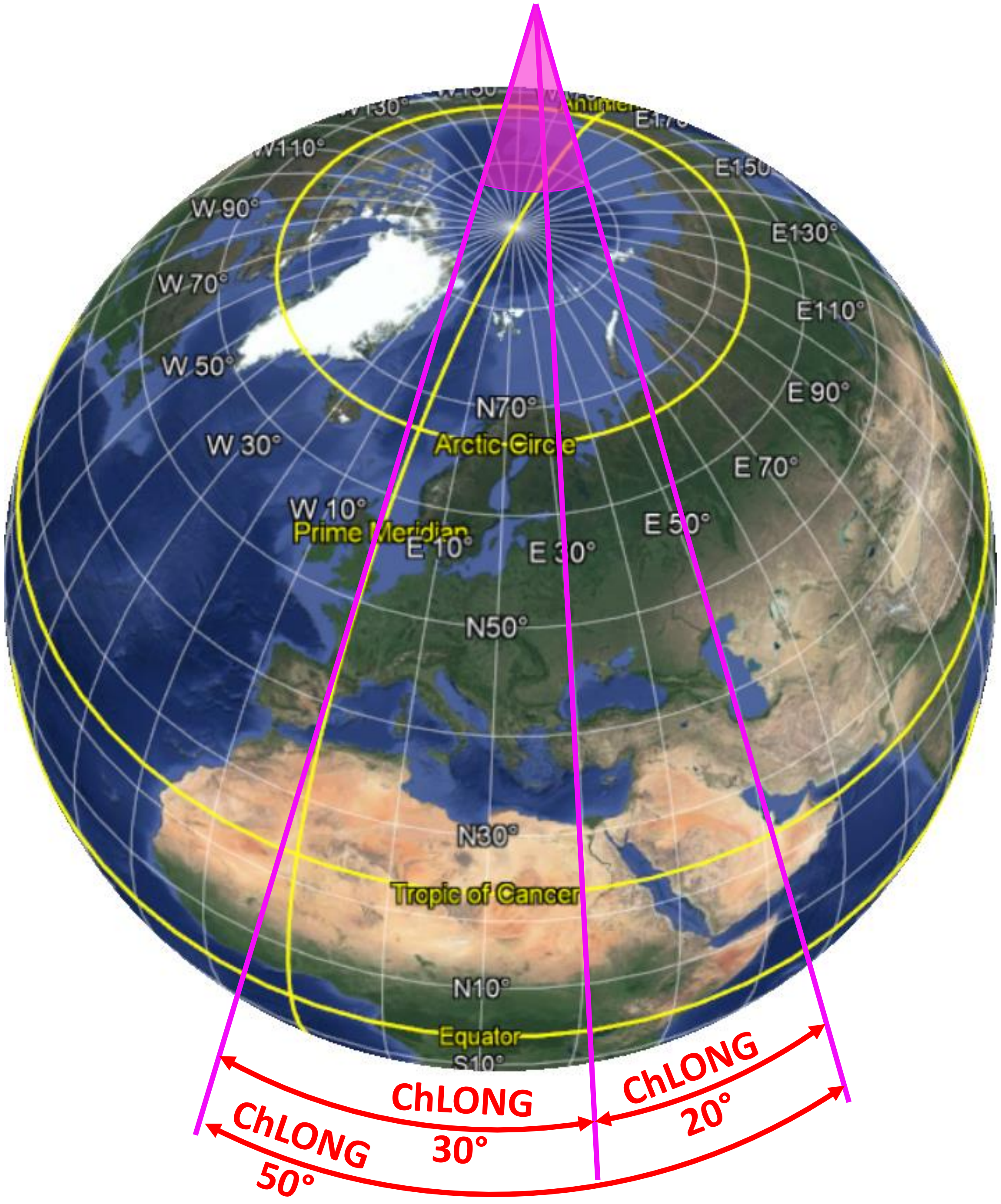
For example, the Prime Meridian (0°E/W) and the meridian 30°E, are separated by 30° and they converge by less than 30°.

If we compare, the Prime Meridian (0°E/W) and the meridian 50°E, are separated by 50° and they converge by less than 50°. However, the convergency is more than the one between the Prime Meridian (0°E/W) and the meridian 30°E.

As well, the meridian 30°E and the meridian 50°E, are separated by 20° and they converge by less than 20°, However, the convergency is less than the one between the Prime Meridian (0°E/W) and the meridian 30°E.

So we can say that the convergency at mid-latitudes depends on the change of longitudes, although it depends also on a **convergency factor 'n'** which makes the convergency less than at the pole.

Convergency = ChLONG x 'n'



When we observe the meridians from the Equator, we can see that they don't converge at all, despite the change of longitudes in between.

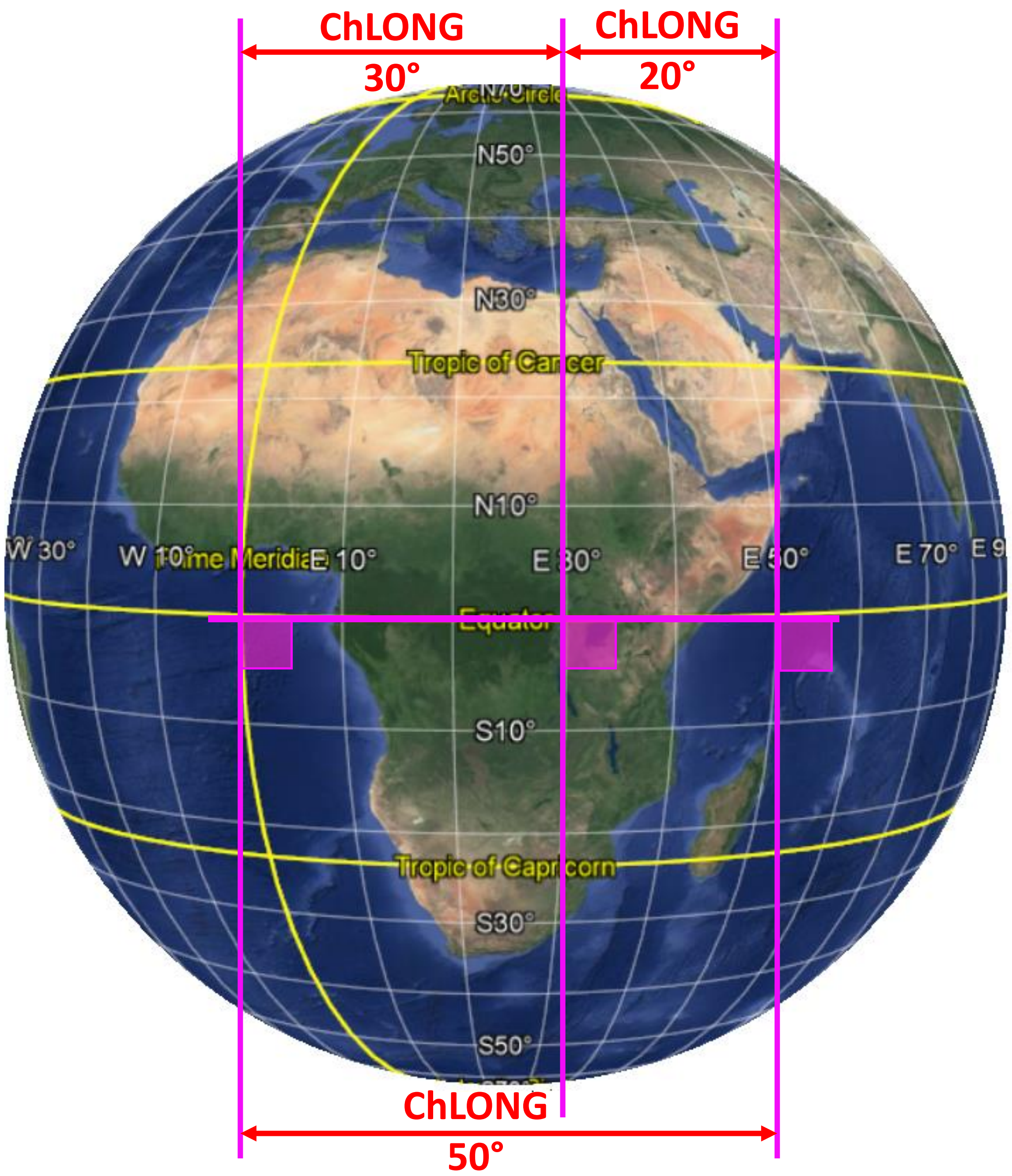
For example, the Prime Meridian (0°E/W) and the meridian 50°E, are separated by 50° and they don't converge.

The same, the Prime Meridian (0°E/W) and the meridian 30°E, are separated by 30° and they don't converge.

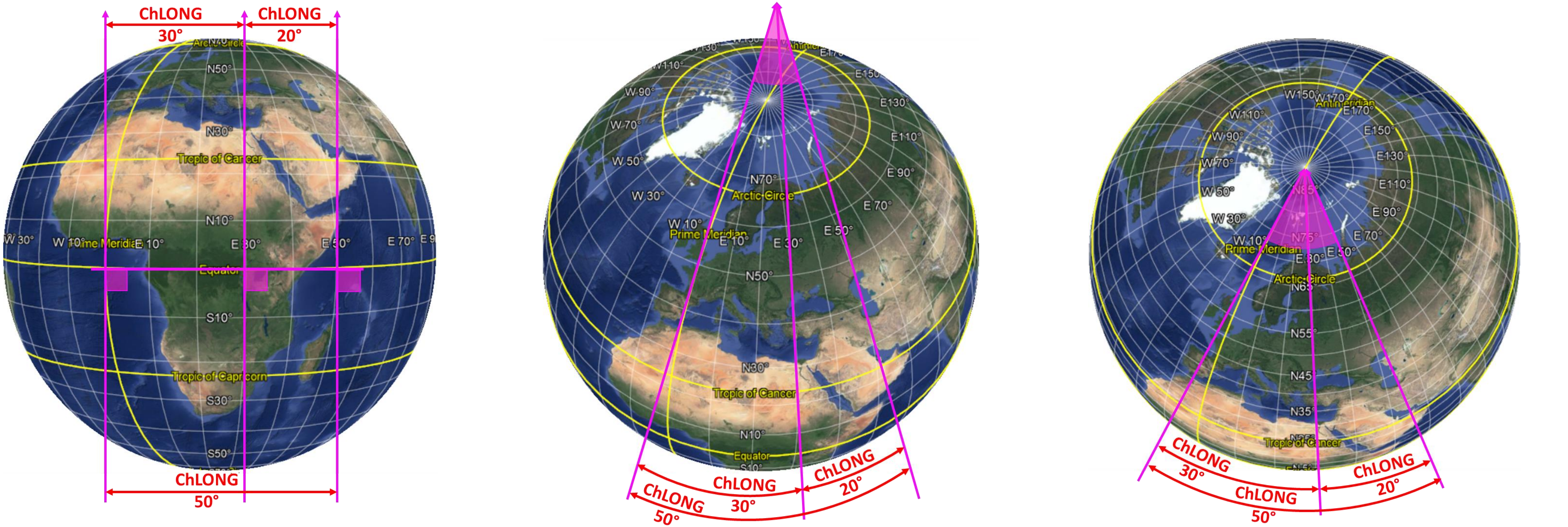
As well, the meridian 30°E and the meridian 50°E, are separated by 20° and they don't converge.

So we can say that the convergency at the Equator is 0°.

Convergency = 0°



We saw that the convergency depends on the change of longitudes in between, as well on the latitudes where the convergency factor “n” is different (no Convergency at the Equator and the Convergency is the ChLONG at the poles).



Convergency = 0	Convergency = ChLONG x *‘n’*	Convergency = ChLONG
Convergency = ChLONG x0	Convergency = ChLONG x *‘n’*	Convergency = ChLONG x1

We saw that the **convergency factor ‘n’** varies between 0 at the Equator (0°) and 1 at the Poles (90°), so we can say :

Convergency = ChLONG x (function of latitude = [0;1])

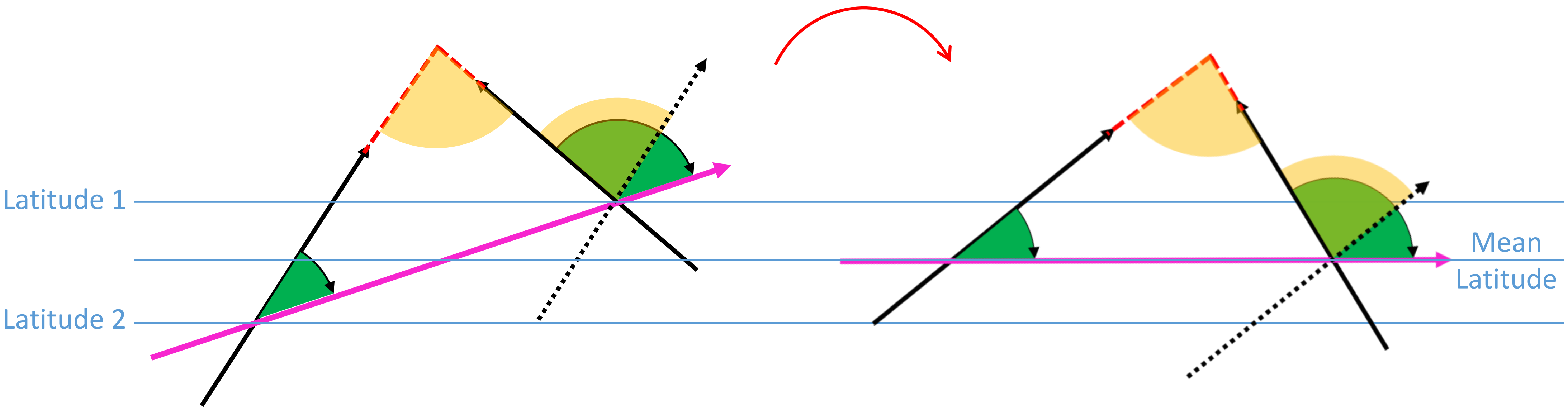
From which we can establish the following equation:

Convergency = ChLONG x sin LAT

So the convergency depends on the change of longitudes (chLONG) and latitude.

Convergency = ChLONG x sin LAT

However this equation is applicable for longitudes compared at the same latitude. When two different latitudes are compared, we can used the average latitude or mean latitude to find the approximate convergency (assumed correct).



So we can modify the equation to obtain:

Convergency = ChLONG x sin Mean LAT

Let's see how we can apply this equation:

Convergency = ChLONG x sin Mean LAT

Eg*. The initial Great Circle track is 060 from A (50°N 30°W) to B (60°N 50°W), what is the final Great Circle Track at B?*

We know that the route between two point will change by the convergency of the meridians at these two points.

Convergency [A/B] = ChLONG [A/B] x sin Mean LAT
Convergency [A/B] = 20° x sin 55°
Convergency [A/B] ≈ 16°

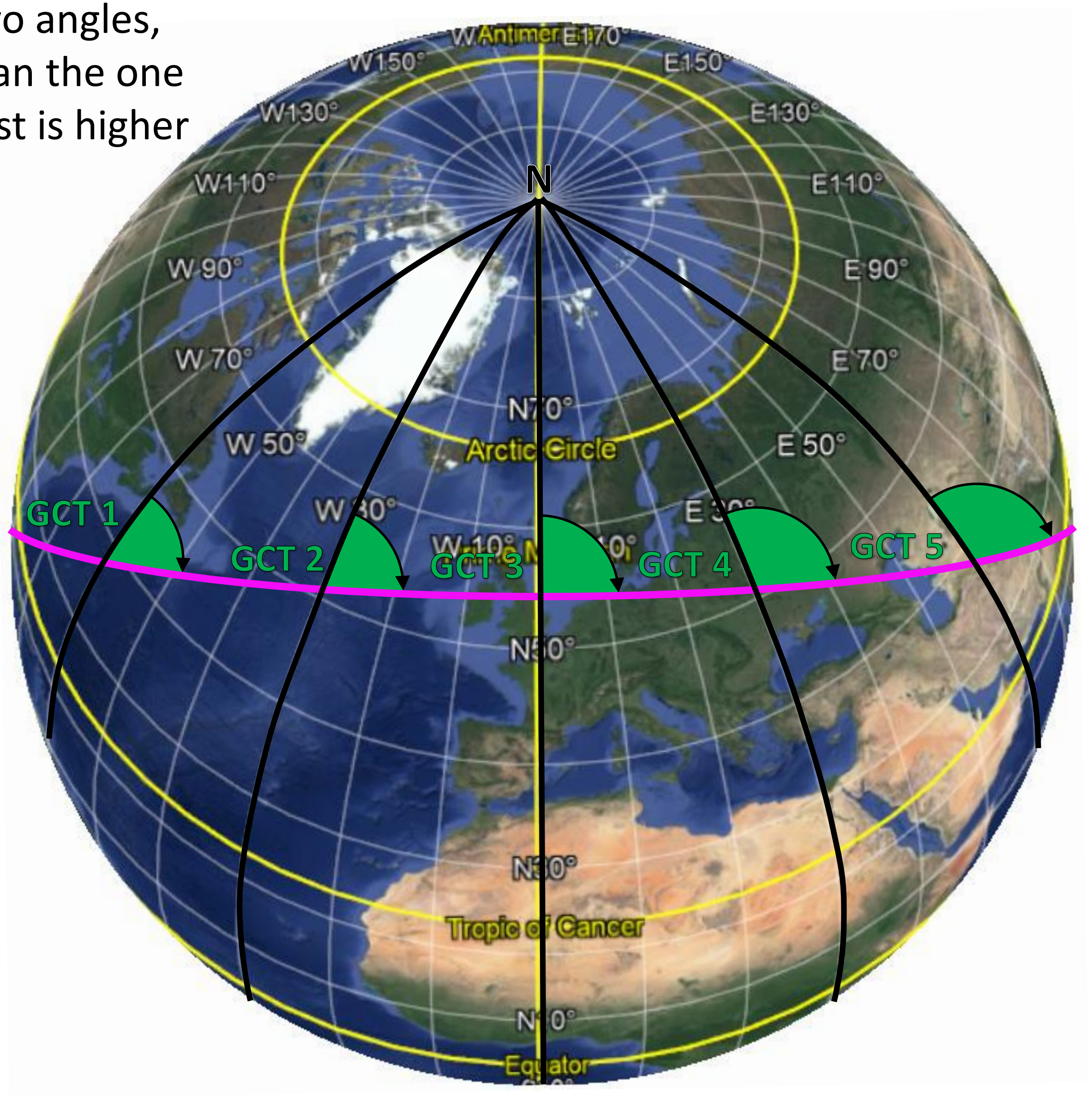
Now we know that the route between these two points will change by 16°. Only one question:

→ The Great Circle Track at B, will it increase by 16° or decrease by 16°?

So now let's see how the Great Circle track increases and decreases.

Northern Hemisphere

When we observe the Great Circle Track (GCT),
We see that, when we compare two angles,
the one on the West is less that than the one
on the East, and the one on the East is higher
than the one on the West.



WEST POSITION
GCT is lower
(DECREASES)

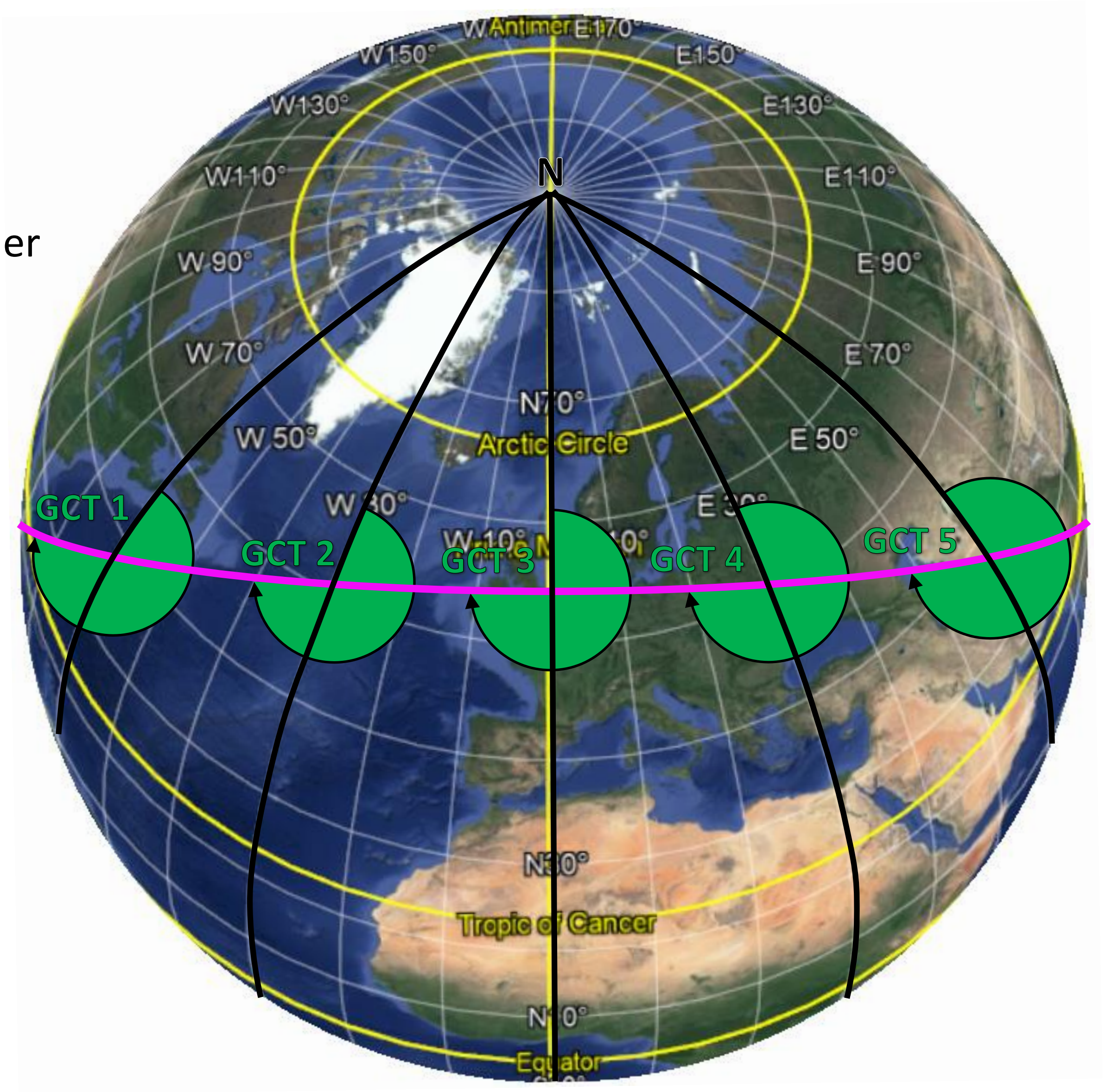
EAST POSITION
GCT is higher
(INCREASES)

W| GTC 1 < GCT 2 < GCT 3 < GCT 4 < GCT 5 |E

Northern Hemisphere

This is applicable no matter whether is westerly or easterly.

WEST POSITION
GCT is lower
(DECREASES)

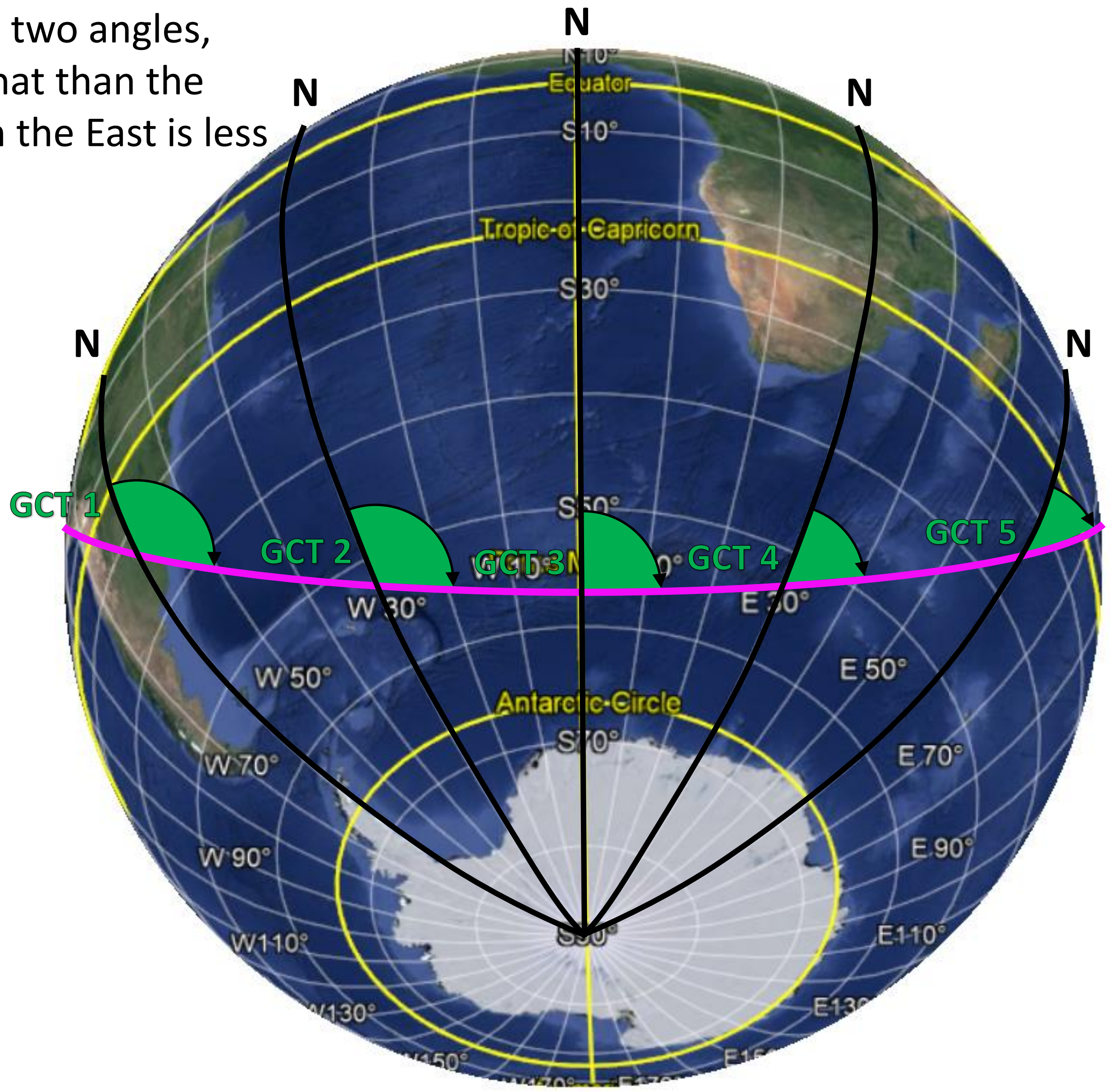


EAST POSITION
GCT is higher
(INCREASES)

W| GTC 1 < GCT 2 < GCT 3 < GCT 4 < GCT 5 |E

Southern Hemisphere

When we observe the Great Circle Track (GCT), We see that, when we compare two angles, the one on the West is higher that than the one on the East, and the one on the East is less than the one on the West.



WEST POSITION
GCT is higher
(INCREASES)

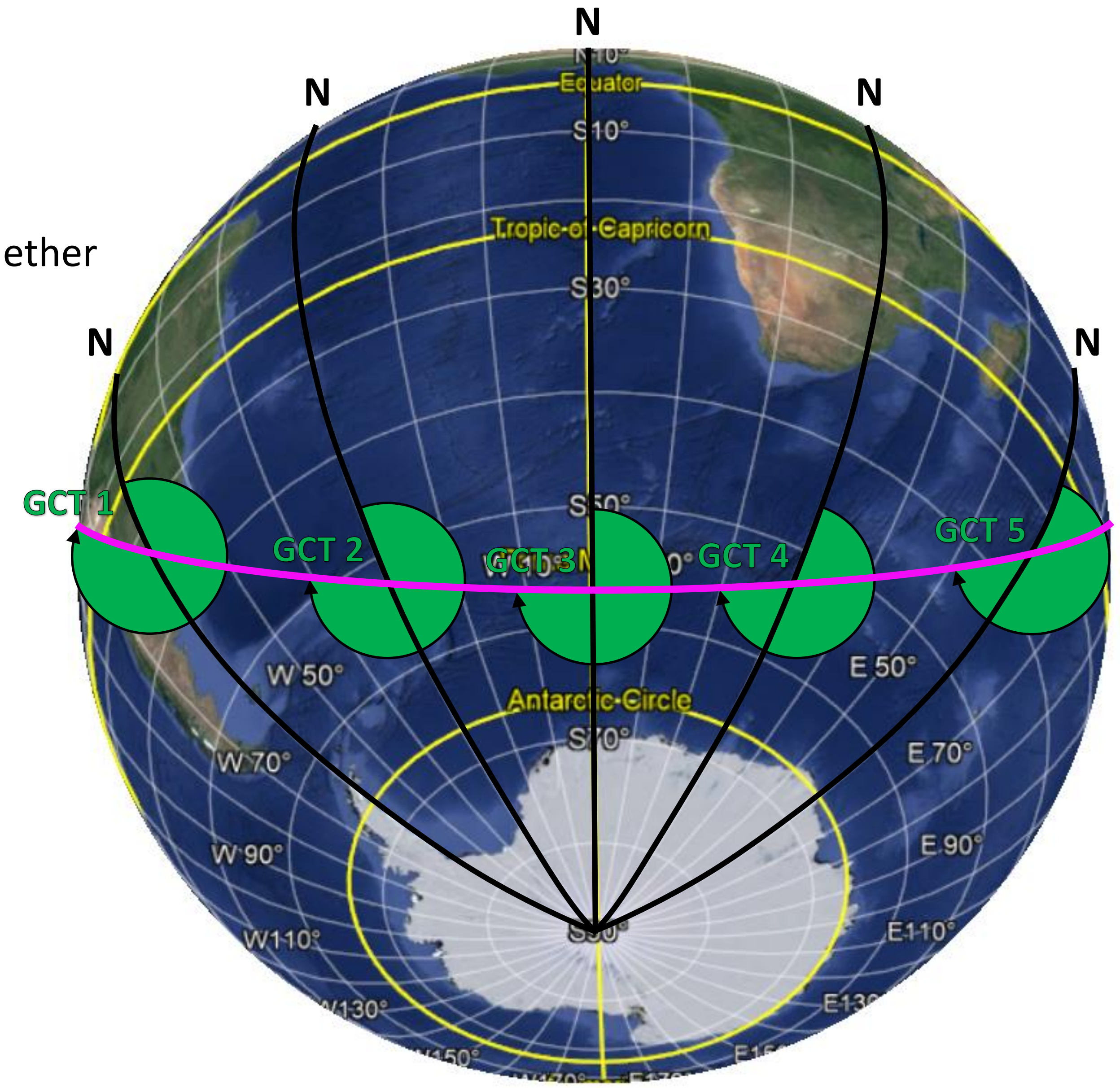
EAST POSITION
GCT is lower
(DECREASES)

W| GTC 1 > GCT 2 > GCT 3 > GCT 4 > GCT 5 |E

Southern Hemisphere

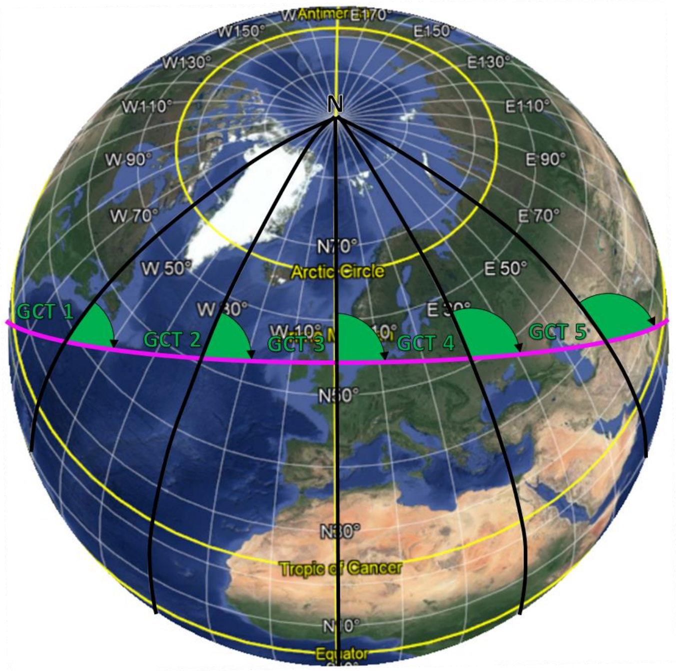
This is applicable no matter whether is westerly or easterly.

WEST POSITION
GCT is higher
(INCREASES)



EAST POSITION
GCT is lower
(DECREASES)

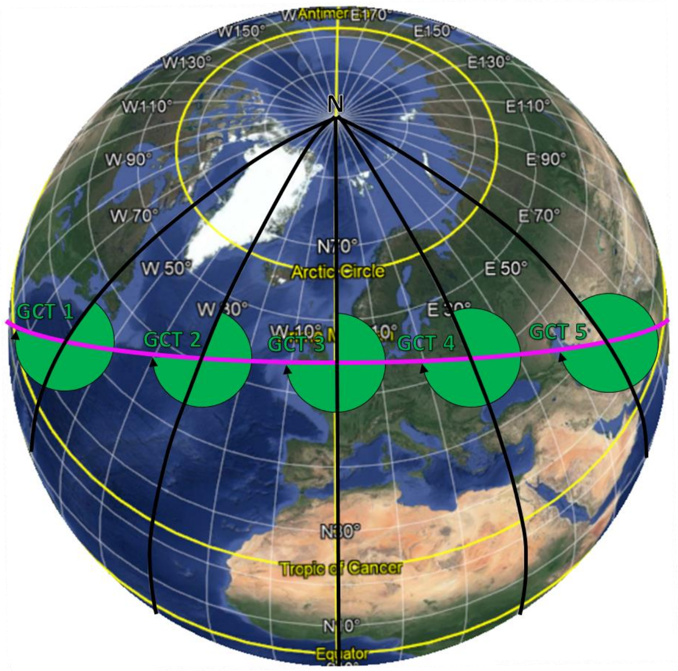
W| GTC 1 > GCT 2 > GCT 3 > GCT 4 > GCT 5 |E

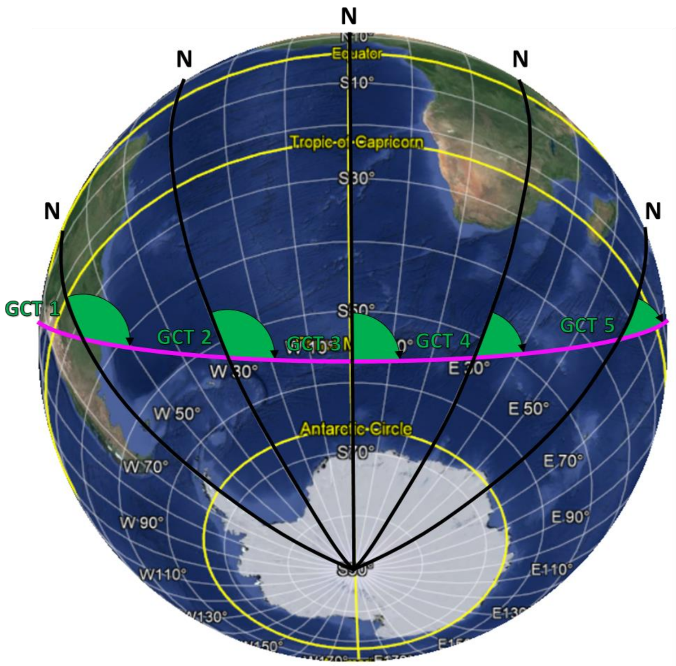


We saw that in the Northern Hemiphere, the GCT on the West positions <u>D</u>ecreases, and the GCT on the East positions <u>I</u>ncreases.

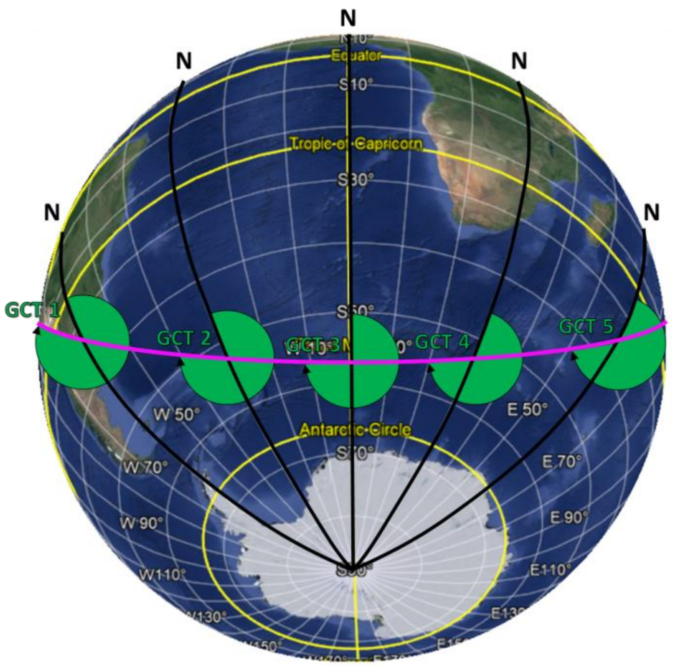
Nevertheless, in the Southern Hemiphere, the GCT on the West positions <u>I</u>ncreases, and the GCT on the East positions <u>D</u>ecreases.

We can use the table below to remember this concept: **DIID**





	WEST POSITION	**EAST POSITION**
Northern Hemisphere	**D**	**I**
Southern Hemisphere	**I**	**D**



Let's see how we can apply this equation:

Convergency = ChLONG x sin Mean LAT

Eg. *The initial Great Circle track is 060 from A (50°N 30°W) to B (60°N 10°W), what is the final Great Circle Track at B?*

We know that the route between two point will change by the convergency of the meridians at these two points.

Convergency [A/B] = ChLONG [A/B] x sin Mean LAT
Convergency [A/B] = 20° x sin 55°
Convergency [A/B] ≈ 16°

Now we know that the route between these two points will change by 16°. Only one question:

→ The Great Circle Track at B, will it increase by 16° or decrease by 16°?

So now let's see how the Great Circle track increases and decreases.

D	I
I	D

Since the location of the point B is East of the point A, and we are in the Northern Hemisphere, so the GCT at B increases compares to the GCT at A, and it will increase by the convergency between A and B.

Therefore, the GCT at B is 076°

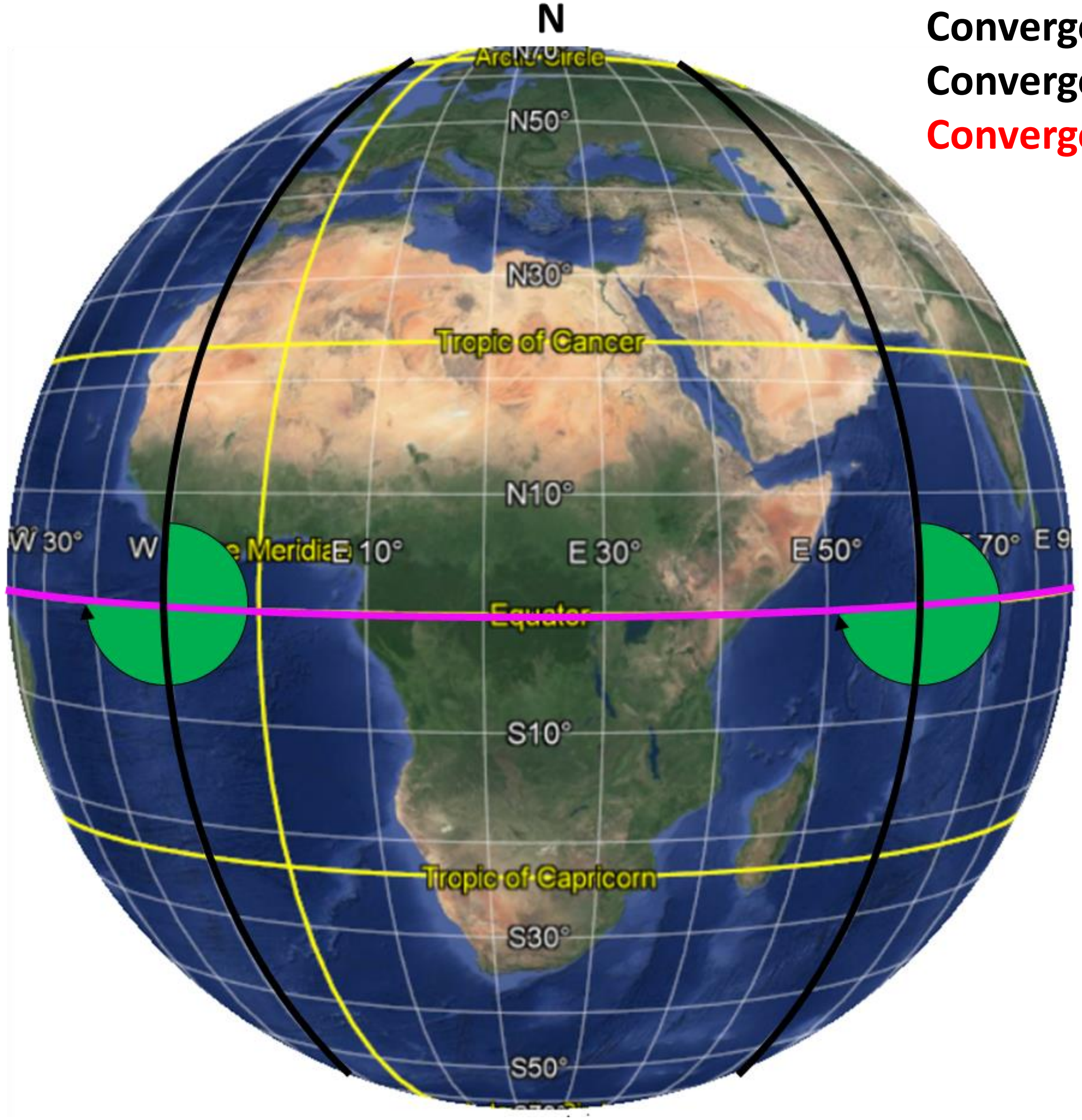
Equator

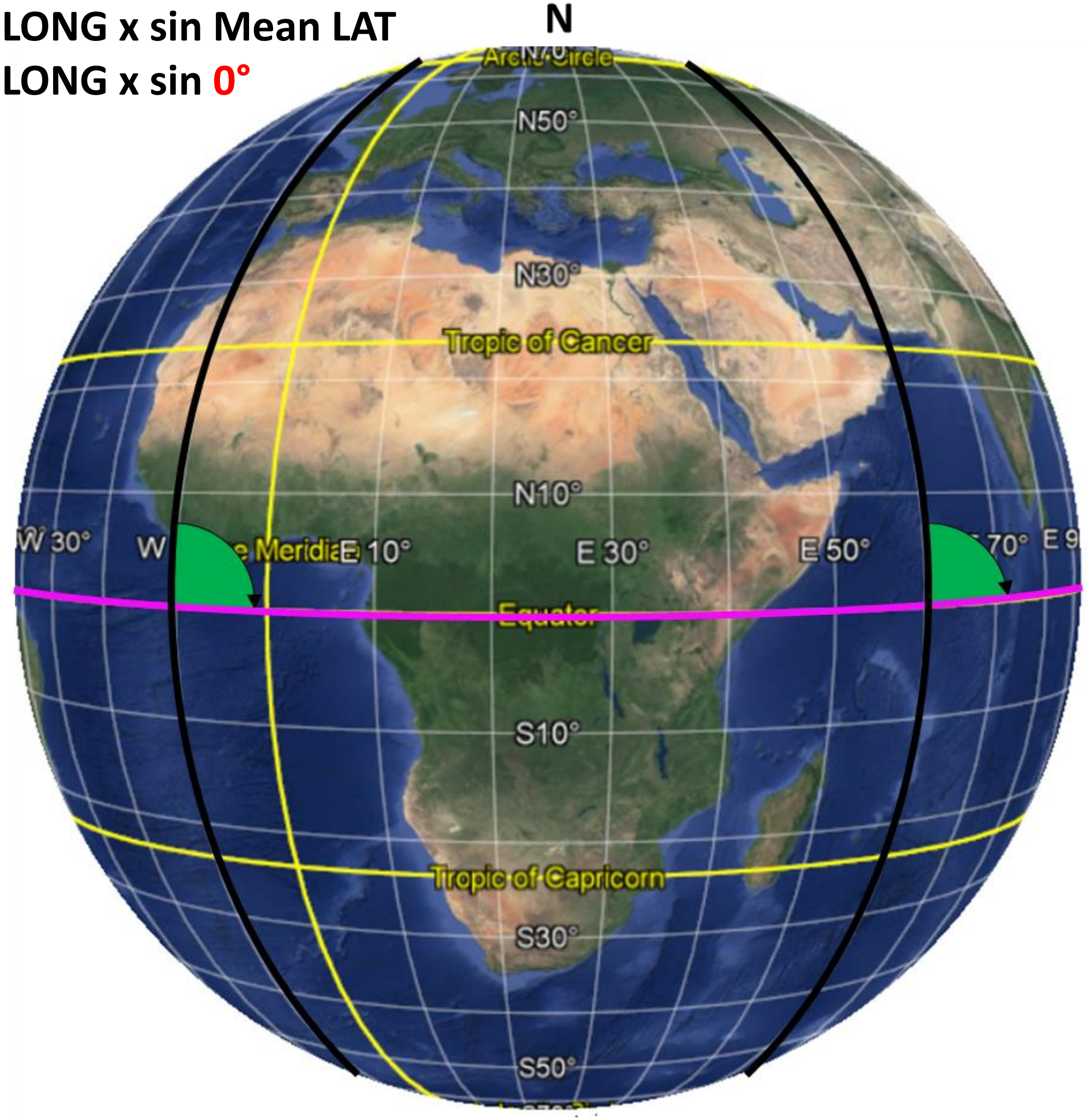
The Great Circle Track (GCT), between positions at the Equator is exactly East or West direction (**090°** or **270°**), and it remains the same since at the Equator the meridians don't converge.

Convergency = ChLONG x sin Mean LAT

Convergency = ChLONG x sin 0°

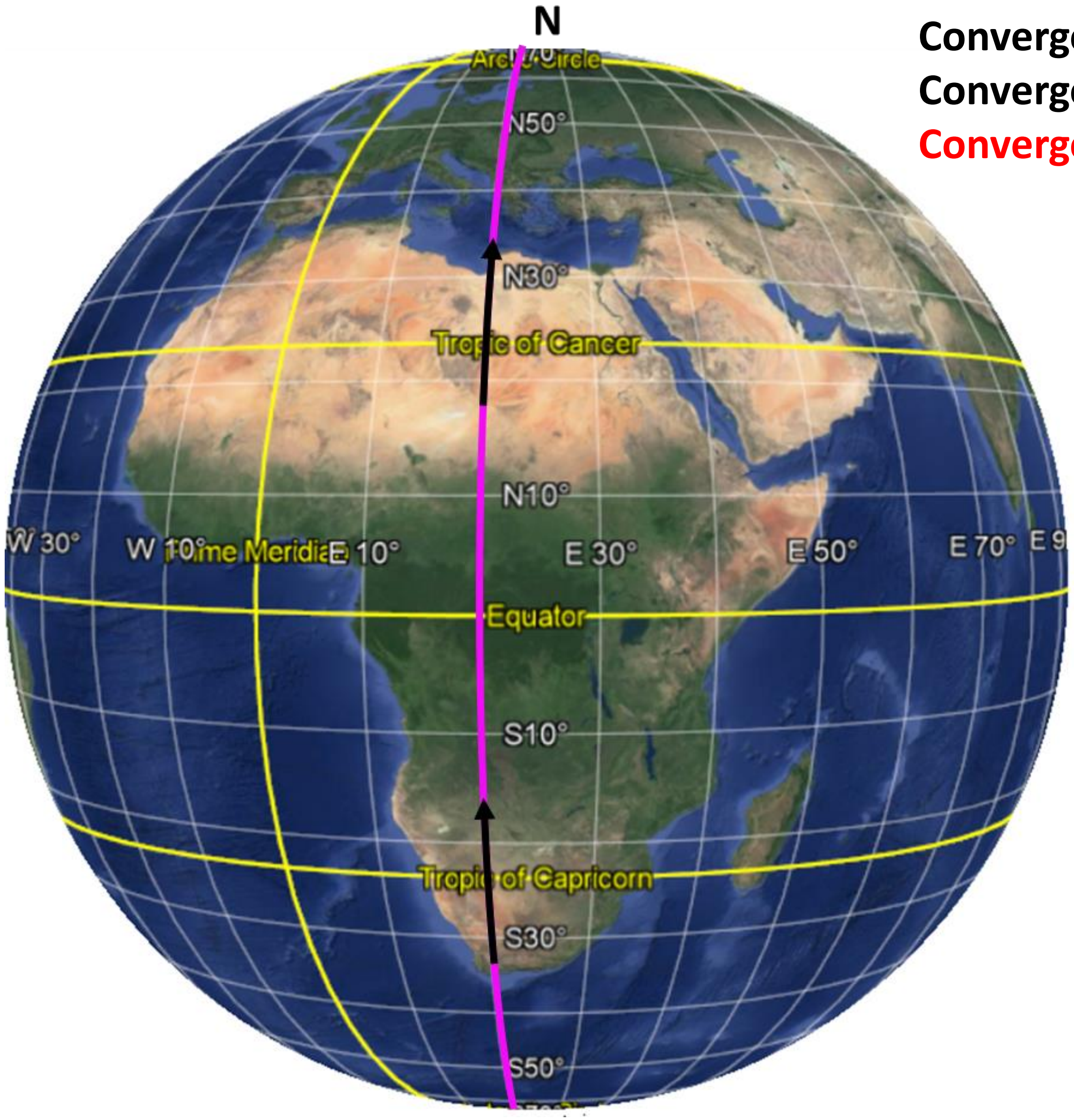
Convergency = 0°





Same Meridian

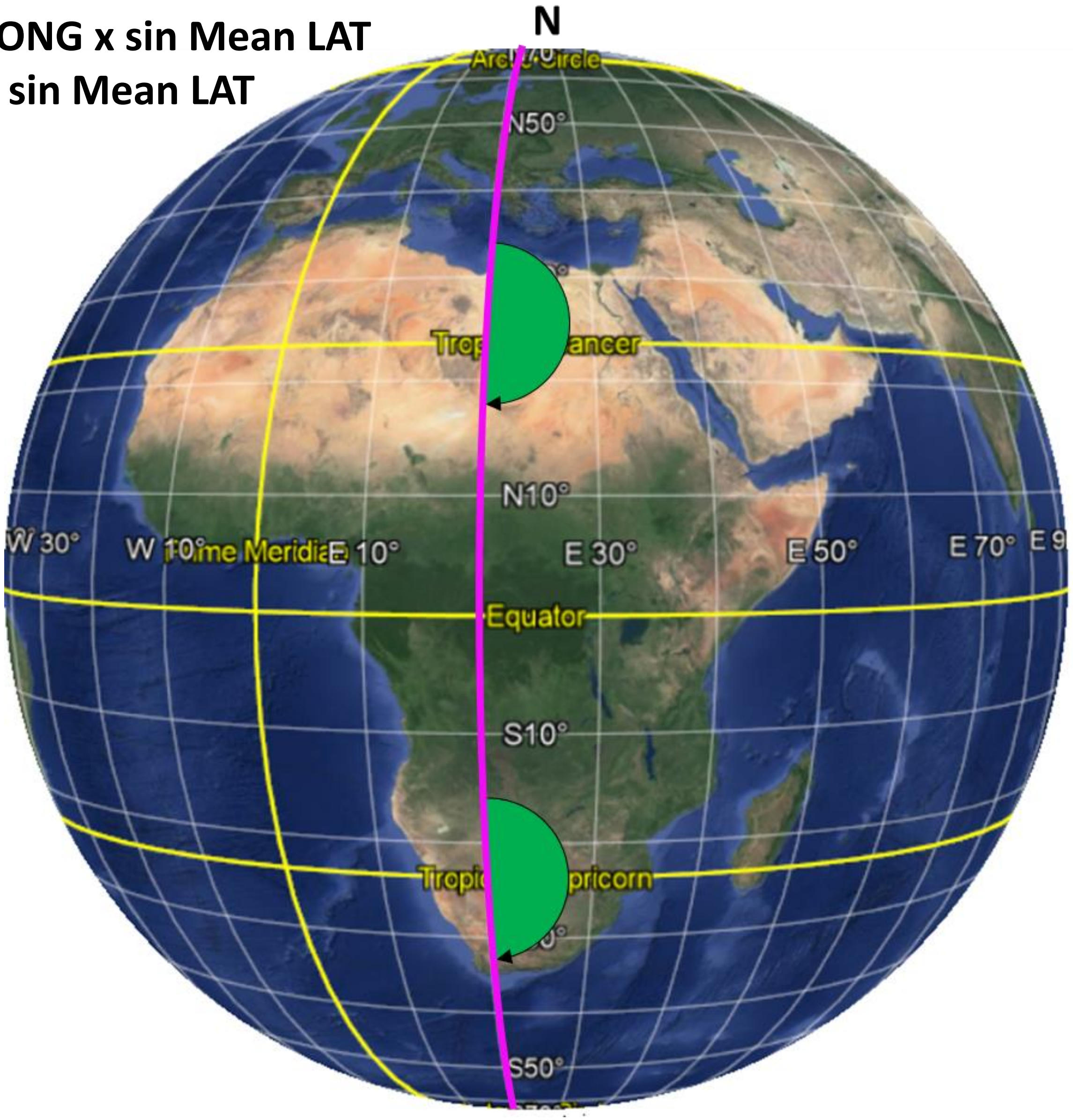
The Great Circle Track (GCT), between positions on the same meridian is exactly North or South direction (**360°** or **180°**), and it remains the same since there is no change of longitude.



Convergency = ChLONG x sin Mean LAT

Convergency = 0° x sin Mean LAT

Convergency = 0°

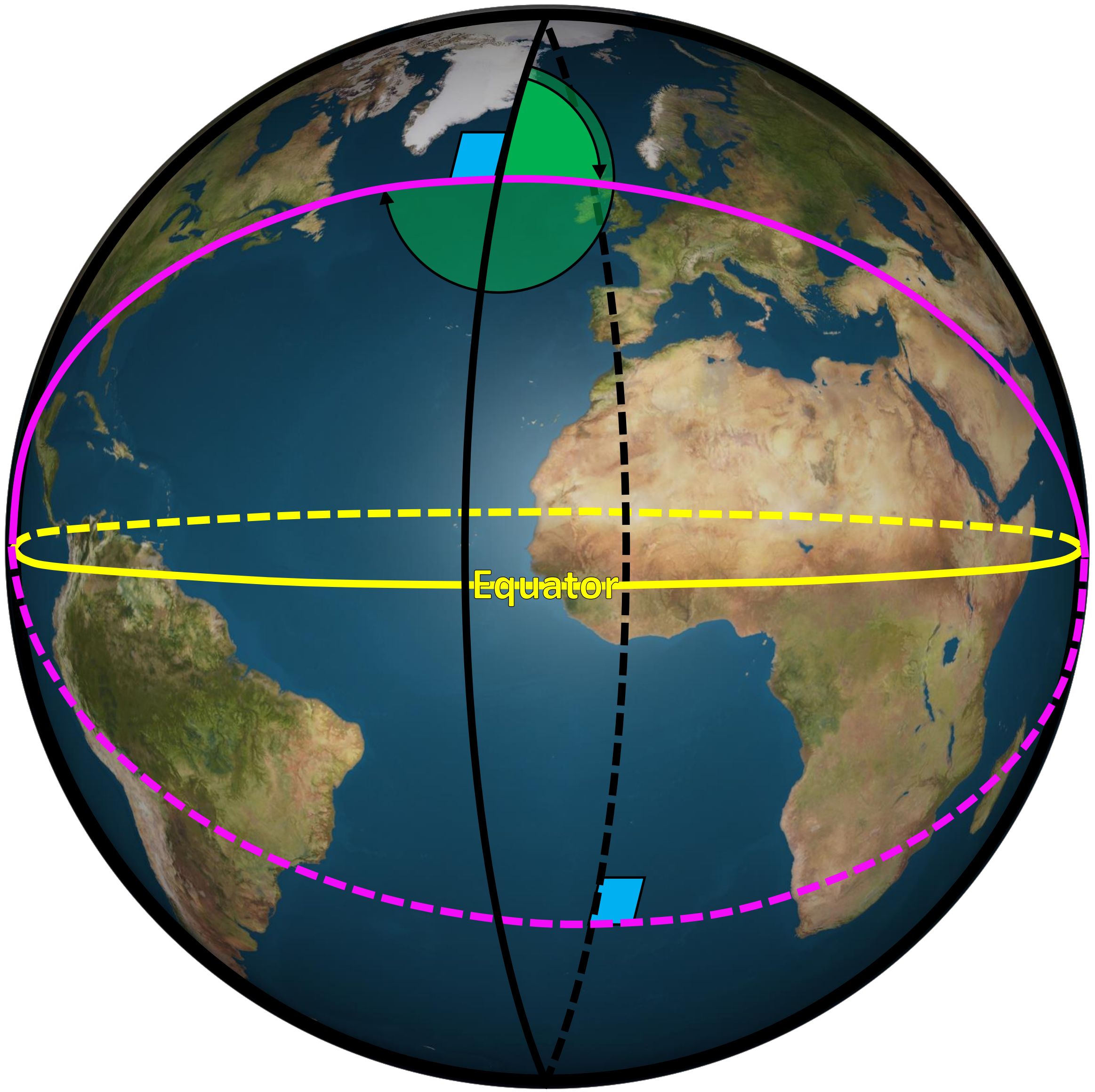


Facts to know…

Fact 1

At the vertices (highest latitude reached by a Great Circle), the Great Circle cuts the meridian at 90° angle. (Except when the Great Circle is the Equator or a Meridian)

This means that, **when passing the highest latitude, the Great Circle Track is 090° or 270°**.
(unless if the initial or final point is at the highest latitude)



Fact 2

For a given convergency between two points. The Great Circle Track at a fraction of route between these two point will change by the same fraction of the convergency in between.

Eg. If the convergency between two points is 40°, it means that the Great Circle Track in between will change by 40°.

→ Halfway, the Great Cricle Track will change by ½ of the convergency, so by 20°

→ At the forth of the route, the Great Circle Track will change by ¼ of the convergency, so 10°

→ At the eighth of the route, the Great Circle Track will change by ⅛ of the convergency, so 5°

Exercises:

a. The initial Great Circle Track from A (45°N 050°E) to B (55°N 060°E) is 040°. What is the final Great Circle Track at B?

b. The initial Great Circle Track from C (50°N 010°E) to D (56°N 020°W) is 260°. What is the final Great Circle Track at D?

c. The initial Great Circle Track from E (40°S 140°W) to F (50°S 160°W) is 250°. What is the final Great Circle Track at F?

d. The final Great Circle Track from G (45°N 010°W) to H (55°N 030°E) is 085°. What is the initial Great Circle Track at G?

e. The final Great Circle Track from I (45°S 050°E) to J (55°S 070°E) is 070°. What is the initial Great Circle Track at I?

f. The initial great circle track from K (40°00’N 20°00’W) to L (50°00’N 10°00’E) is 060°. What is the initial Great Circle Track from L to K?

g. The initial great circle track from M (40°00’S 170°00’W) to N (45°00’S 174°00’E) is 250°. What is the initial great circle track from N to M?

h. The initial great circle track from O (36°00’N 15°00E) to P (latitude 42°00N’) is 300° and the final great circle track at P is 295°. (1) What is the longitude of P? (2) What is the approximate great circle track direction at longitude 11°00E?

i. The initial great circle track from Q (28°00’N 008°00’W) to R (32°00N’ 012°00’E) is 070°. What is the Great Circle Track when passing the Prime Meridian?

The correction is in the next page…

Exercise:

a. The initial Great Circle Track from A (45°N 050°E) to B (55°N 060°E) is 040°. What is the final Great Circle Track at B?

We know that the route between two points will change by the convergency of the meridians at these two points.

Convergency [A/B] = ChLONG [A/B] x sin Mean LAT

Convergency [A/B] = 10° x sin 50°

Convergency [A/B] ≈ 8°

Now we know that the route between these two points will change by 8°:

So now let's see how the Great Circle track increases and decreases.

D	I
I	D

Since the location of the point B is East of the point A, and we are in the Northern Hemisphere, so the GCT at B increases compared to the GCT at A, and it will increase by the convergency between A and B.

Therefore, the GCT at B is 048°

Exercise:

b. The initial Great Circle Track from C (50°N 010°E) to D (56°N 020°W) is 260°. What is the final Great Circle Track at D?

We know that the route between two points will change by the convergency of the meridians at these two points.

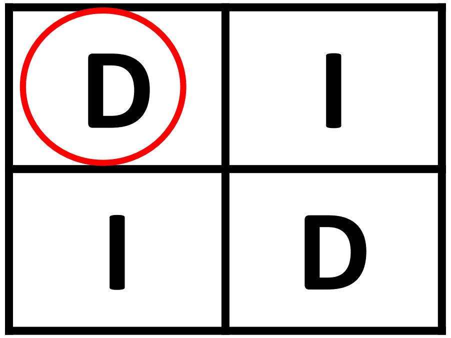
Convergency [C/D] = ChLONG [C/D] x sin Mean LAT

Convergency [C/D] = 30° x sin 53°

Convergency [C/D] ≈ 24°

Now we know that the route between these two points will change by 24°:

So now let’s see how the Great Circle track increases and decreases.



Since the location of the point D is West of the point C, and we are in the Northern Hemisphere, so the GCT at D decreases compared to the GCT at C, and it will increase by the convergency between C and D.

Therefore, the GCT at D is 236°

Exercise:

c. The initial Great Circle Track from E (40°S 140°W) to F (50°S 160°W) is 250°. What is the final Great Circle Track at F?

We know that the route between two points will change by the convergency of the meridians at these two points.

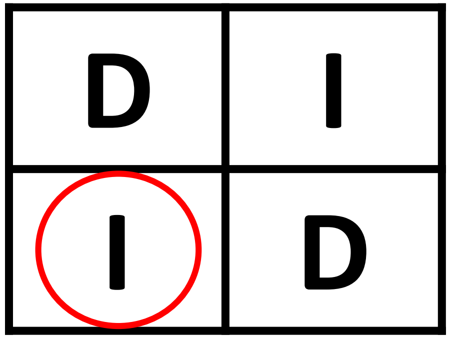
Convergency [E/F] = ChLONG [E/F] x sin Mean LAT

Convergency [E/F] = 20° x sin 45°

Convergency [E/F] ≈ 14°

Now we know that the route between these two points will change by 14°:

So now let's see how the Great Circle track increases and decreases.



Since the location of the point F is West of the point E, and we are in the Southern Hemisphere, so the GCT at F increases compared to the GCT at E, and it will increase by the convergency between E and F.

Therefore, the GCT at F is 264°

Exercise:

d. The final Great Circle Track from G (45°N 010°W) to H (55°N 030°E) is 085°. What is the initial Great Circle Track at G?

We know that the route between two points will change by the convergency of the meridians at these two points.

Convergency [G/H] = ChLONG [G/H] x sin Mean LAT

Convergency [G/H] = 40° x sin 50°

Convergency [G/H] ≈ 31°

Now we know that the route between these two points will change by 31°:

So now let's see how the Great Circle track increases and decreases.

D	I
I	D

Since the location of the point G is West of the point H, and we are in the Northern Hemisphere, so the GCT at G decreases compared to the GCT at H, and it will increase by the convergency between G and H.

Therefore, the GCT at G is 054°

Exercise:

e. The final Great Circle Track from I (45°S 050°E) to J (55°S 070°E) is 070°. What is the initial Great Circle Track at I?

We know that the route between two points will change by the convergency of the meridians at these two points.

Convergency [I/J] = ChLONG [I/J] x sin Mean LAT

Convergency [I/J] = 20° x sin 50°

Convergency [I/J] ≈ 15°

Now we know that the route between these two points will change by 15°:

So now let’s see how the Great Circle track increases and decreases.

D	I
I	D

Since the location of the point I is West of the point J, and we are in the Southern Hemisphere, so the GCT at I increases compared to the GCT at J, and it will increase by the convergency between I and J.

Therefore, the GCT at I is 085°

Advanced Exercise:

f. The initial great circle track from K (40°00’N 20°00’W) to L (50°00’N 10°00’E) is 060°. What is the initial Great Circle Track from L to K?

We know that the route between two points will change by the convergency of the meridians at these two points.

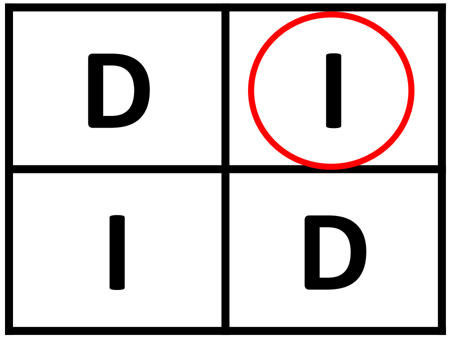
Convergency [K/L] = ChLONG [K/L] x sin Mean LAT

Convergency [K/L] = 30° x sin 45°

Convergency [K/L] ≈ 21°

Now we know that the route between these two points will change by 21°:

So now let’s see how the Great Circle track increases and decreases.



Since the location of the point L is East of the point K, and we are in the Northern Hemisphere, so the GCT at L increases compared to the GCT at K, and it will increase by the convergency between K and L.

However the question is asking the initial Great Circle Track at L when returning to K. So we will simply use the reciprocal of the Great Circle Track at L.

Therefore, the initial GCT from L to K is 261°

Advanced Exercise:

g. The initial great circle track from M (40°00’S 170°00’W) to N (45°00’S 174°00’E) is 250°. What is the initial great circle track from N to M?

We know that the route between two points will change by the convergency of the meridians at these two points.

Convergency [M/N] = ChLONG [M/N] x sin Mean LAT

Convergency [M/N] = 16° x sin 42.5°

Convergency [M/N] ≈ 11°

Now we know that the route between these two points will change by 11°:

So now let’s see how the Great Circle track increases and decreases.

D	I
I	D

Since the location of the point N is West of the point M, and we are in the Southern Hemisphere, so the GCT at N increases compared to the GCT at M, and it will increase by the convergency between M and N.

However the question is asking the initial Great Circle Track at N when returning to M. So we will simply use the reciprocal of the Great Circle Track at N.

Therefore, the initial GCT from N to M is 081°

Advanced Exercise:

h. The initial great circle track from O (36°00’N 15°00E) to P (latitude 42°00N’) is 300° and the final great circle track at P is 295°.

1. What is the longitude of P?
2. What is the approximate great circle track direction at longitude 11°00E?

Answer 1.

- To find the location of the point P, we need to know the change of longitudes between O and P
- We know that the route between two points will change by the convergency of the meridians at these two points.
- From O to P, the Great Circle Track changes by 5°, so the convergency between these two locations is 5°

$$\text{Convergency [O/P]} = \text{ChLONG [O/P]} \times \sin \text{Mean LAT}$$

$$\leftrightarrow \text{ChLONG [O/P]} = \frac{\text{Convergency [O/P}}{\sin \text{Mean LAT}} = \frac{5°}{\sin 39°}$$

ChLONG [O/P] = 8°

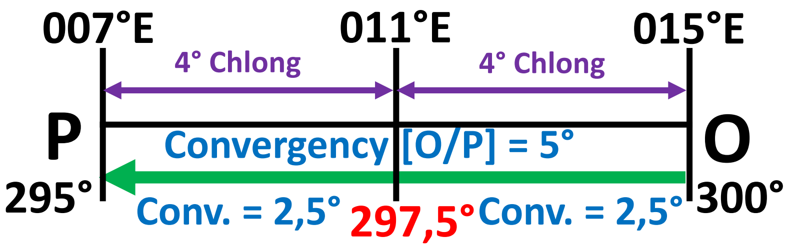
- The meridian of the location P is 8° change of longitudes away from O at the meridian 15°E.
- Given the westerly tracks (300° and 295°), and the route starts at O, so P is West of O (in addition, the route decreases at P compared to O and we are in the Northern Hemisphere)

→ **So the longitude of P is 007°E**

Answer 2.

- The meridian 11°E is halfway between the meridians of the location O and P (4° change of longitude away from O and P)
- So the Great Circle Track at 11°E will change by ½ of the convergency from O and P, so by **2,5°**

→ **So the GCT at the meridian 11°E is 297,5°**



Advanced Exercise:

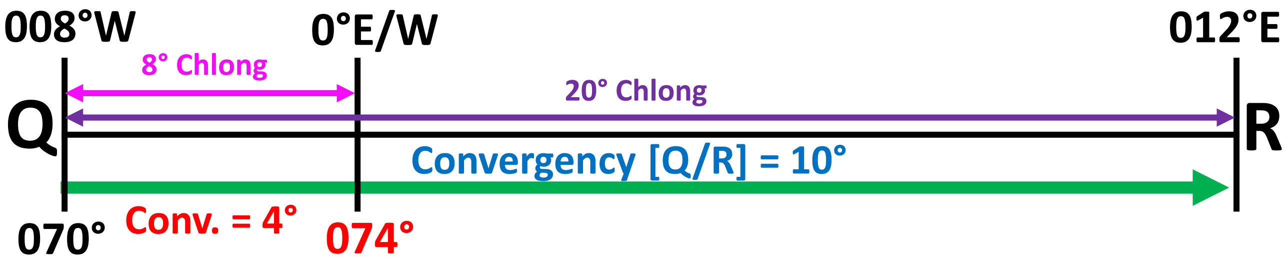
i. The initial great circle track from Q (28°00’N 008°00’W) to R (32°00N’ 012°00’E) is 070°. What is the Great Circle Track when passing the Prime Meridian?

We know that the route between two points will change by the convergency of the meridians at these two points.

Convergency [Q/R] = ChLONG [Q/R] x sin Mean LAT
Convergency [Q/R] = 20° x sin 30°
Convergency [Q/R] = 10°

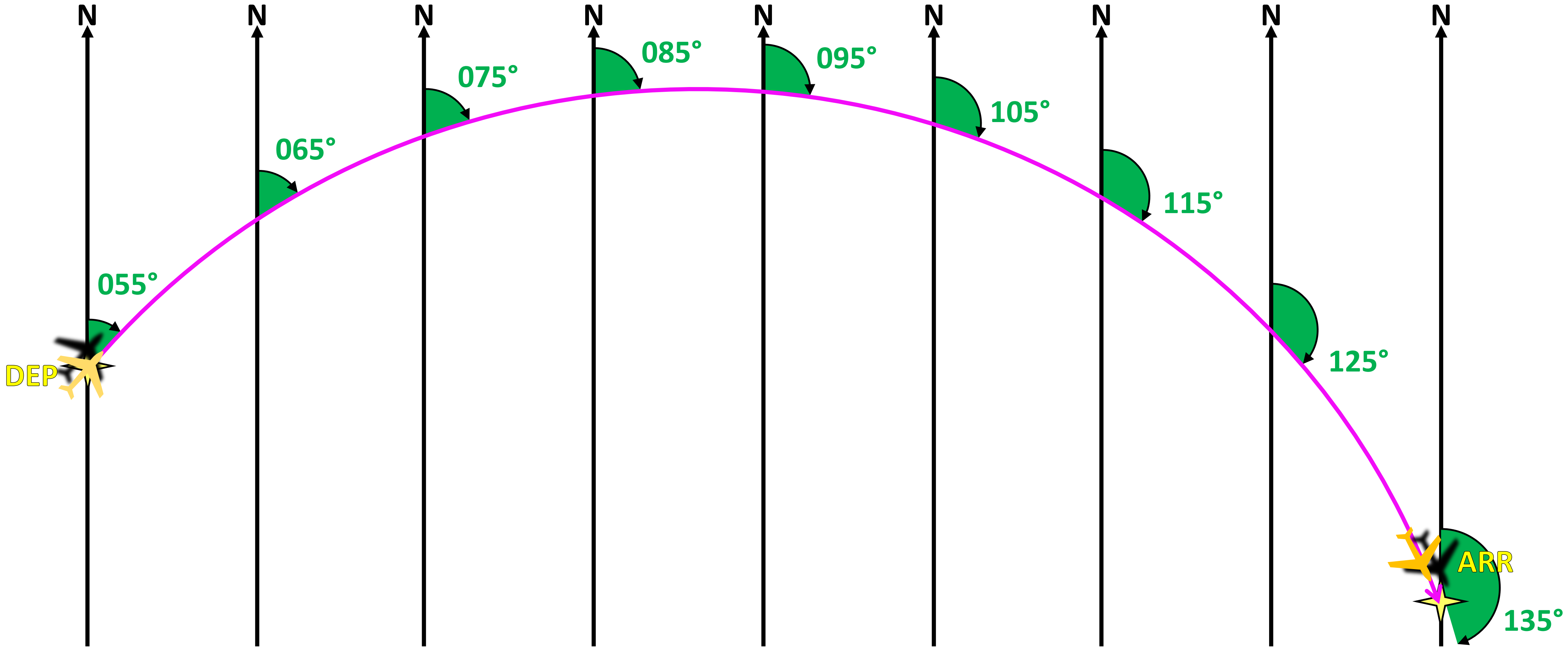
- The Prime meridian is 8° change of longitudes away from the meridian of the location Q, which is ⅖ of the route (8°/20°).
- So the Great Circle Track at the Prime Meridian changes by ⅖ of the convergency between Q and R, so by **4°** (⅖ of 10° = 4°)
- Since the location of the point Q is West of the point R, so the point at ⅖ of the route is East of Q, and we are in the Northern Hemisphere, so the GCT at the point of ⅖ of the route increases compared to the GCT at Q, and it will increase by the ⅖ of the convergency between Q and R.

D	I
I	D



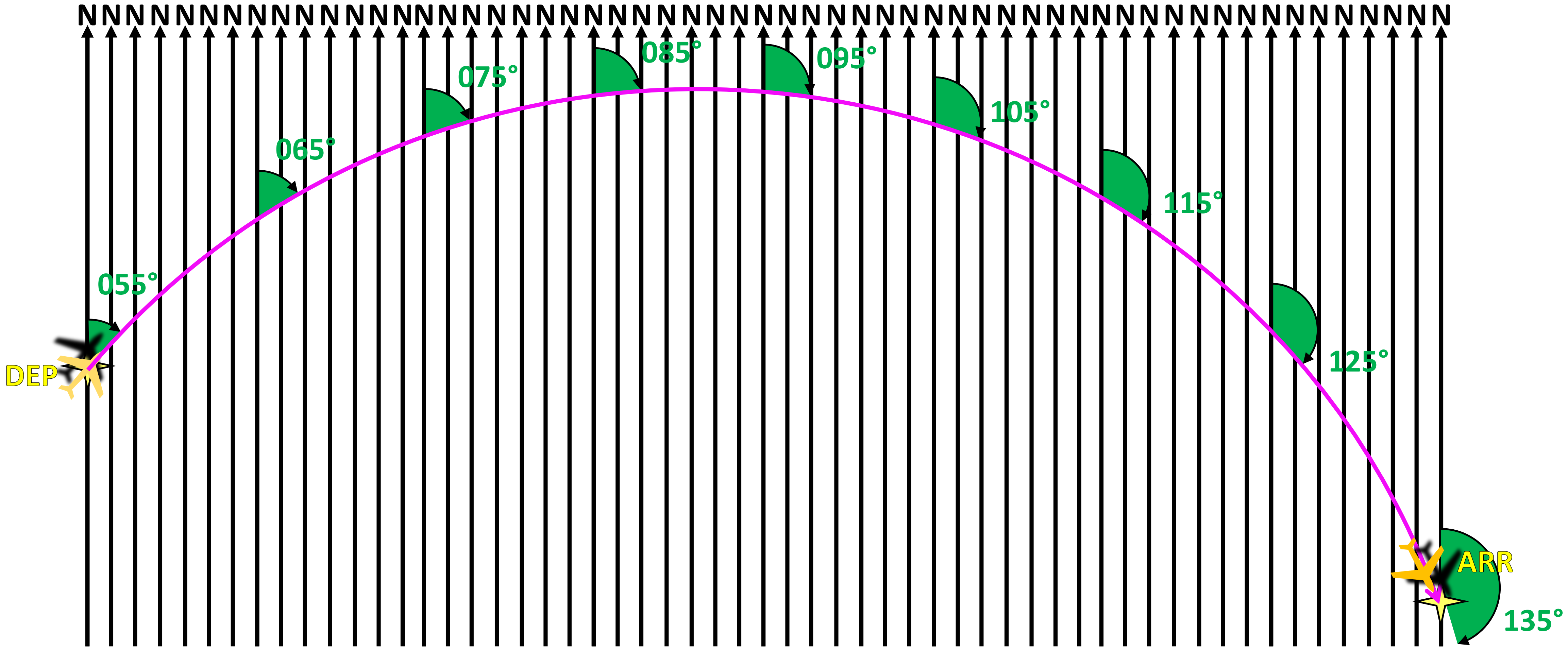
→ **So the GCT when passing the Prime Meridian is 074°**

If we draw the **Great Circle** between two points Departure (DEP) and Arrival (ARR), the route is constantly changing every time we change the meridian. When you navigate and you see your destination is insight, you simply target your destination without the need to watch any navigational instrument, and you will automatically follow the **Great Circle Track** to your destination.



However it is really rare to have the destination insight straight after departure (far destination, IMC, etc.) So we need to know the angle to follow and to maintain on the navigational instruments.
So at which meridian shall we measure the route and how many angles shall we measure ?

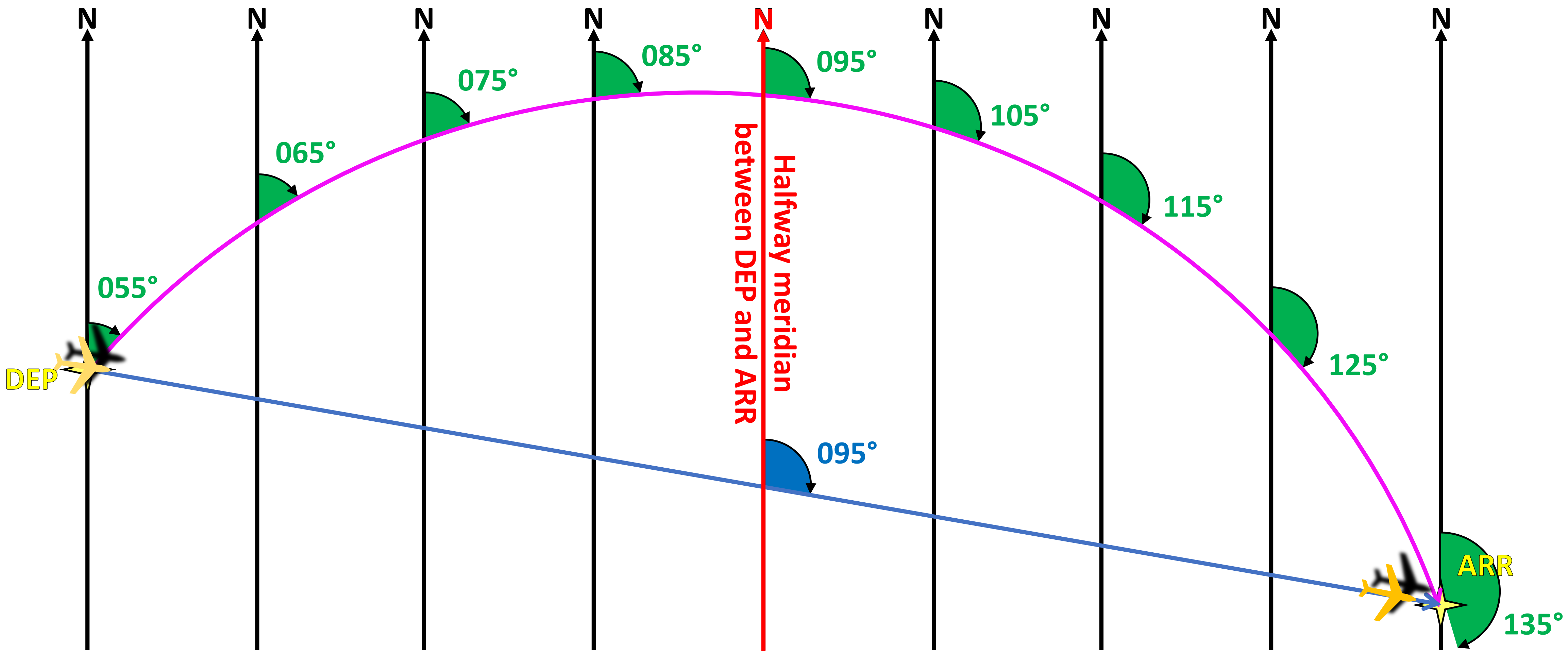
Indeed it exists an infinite number of meridians, where the Great Circle Track is different, which means that you must constantly 'turn' and adjust the track to remain on the Great Circle from Departure to Arrival, which is in fact, not convenient.



So how can we decide after how many change of longitudes shall we measure the Great Circle Track angle?

The easiest would be to draw the **Rhumb Line** from the Departure to the Arrival point. Indeed the **Rhumb Line Track** (**RLT**) is one angle, and navigating with one angle only makes the navigation very easy.

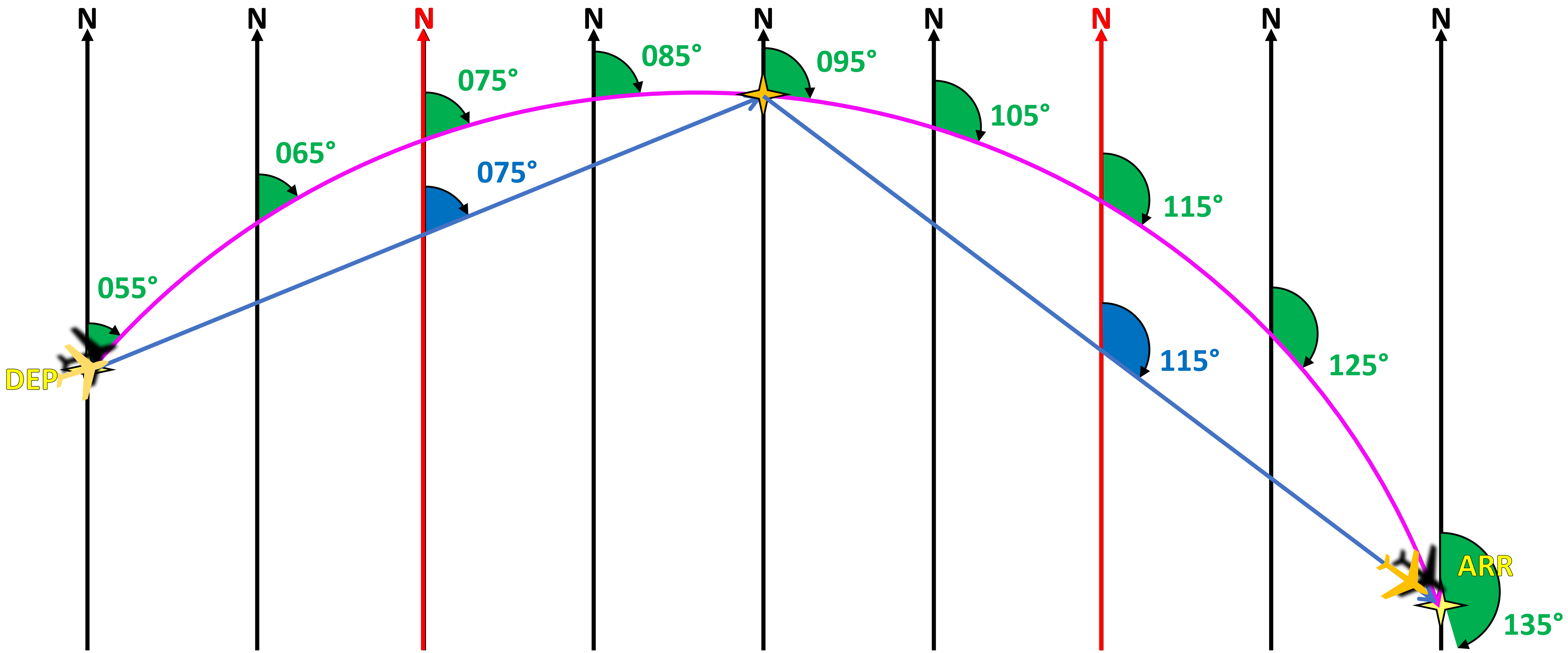
However, the Rhumb Line Track from Departure to Arrival is far from the Great Circle Track, which makes the route much longer.



Notice that the **Rhumb Line Track** from Departure to Arrival is equal to the **Average Great Circle Track** from the Departure to the Arrival point [(055+135)/2=095], or the **Great Circle Track halfway** between the Departure and the Arrival point.

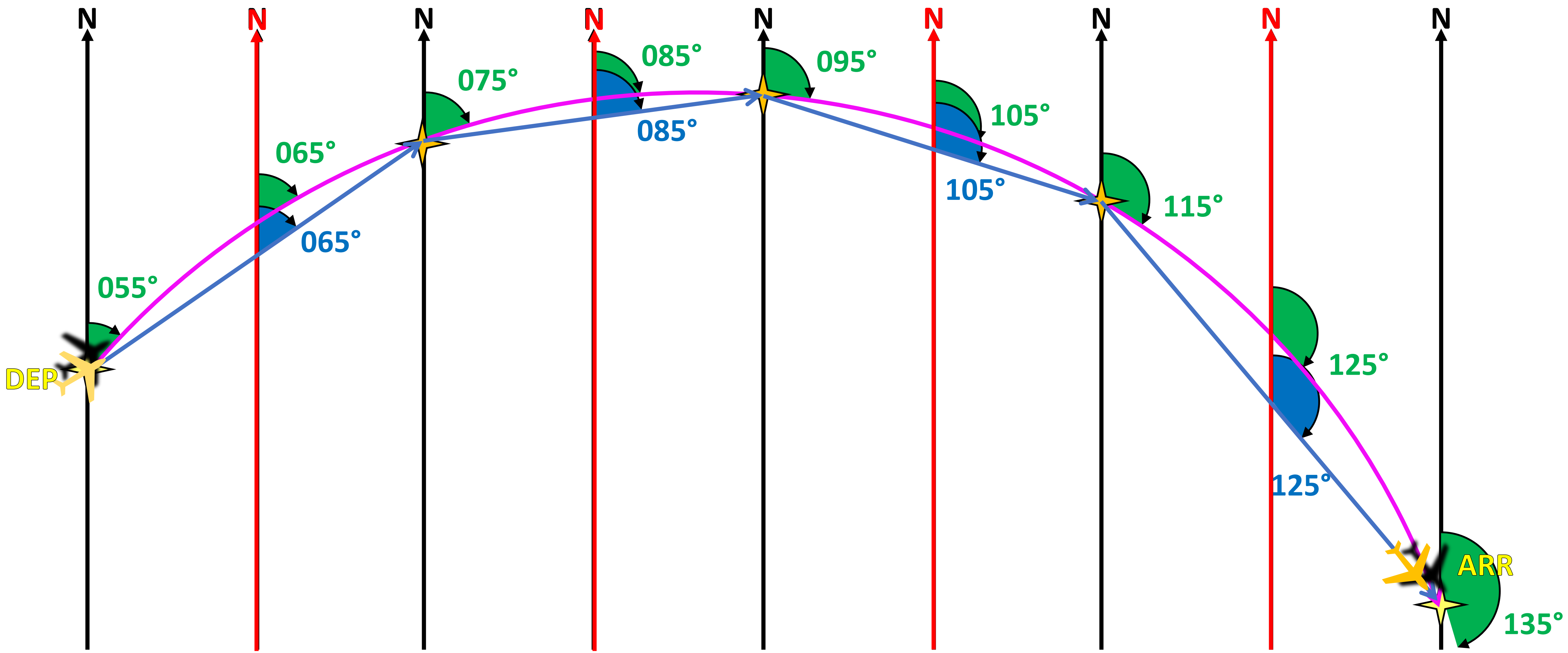
RLT [A/B] = Average GCT [A/B] = GCT halfway [A/B]

To remain closer to the **Great Circle** from Departure to Arrival, we can add a 'waypoint' (WP1) ✦ on the **Great Circle** between the Departure and the Arrival point, and follow the **Rhumb Lines** from DEP to WP1, then from WP1 to ARR.



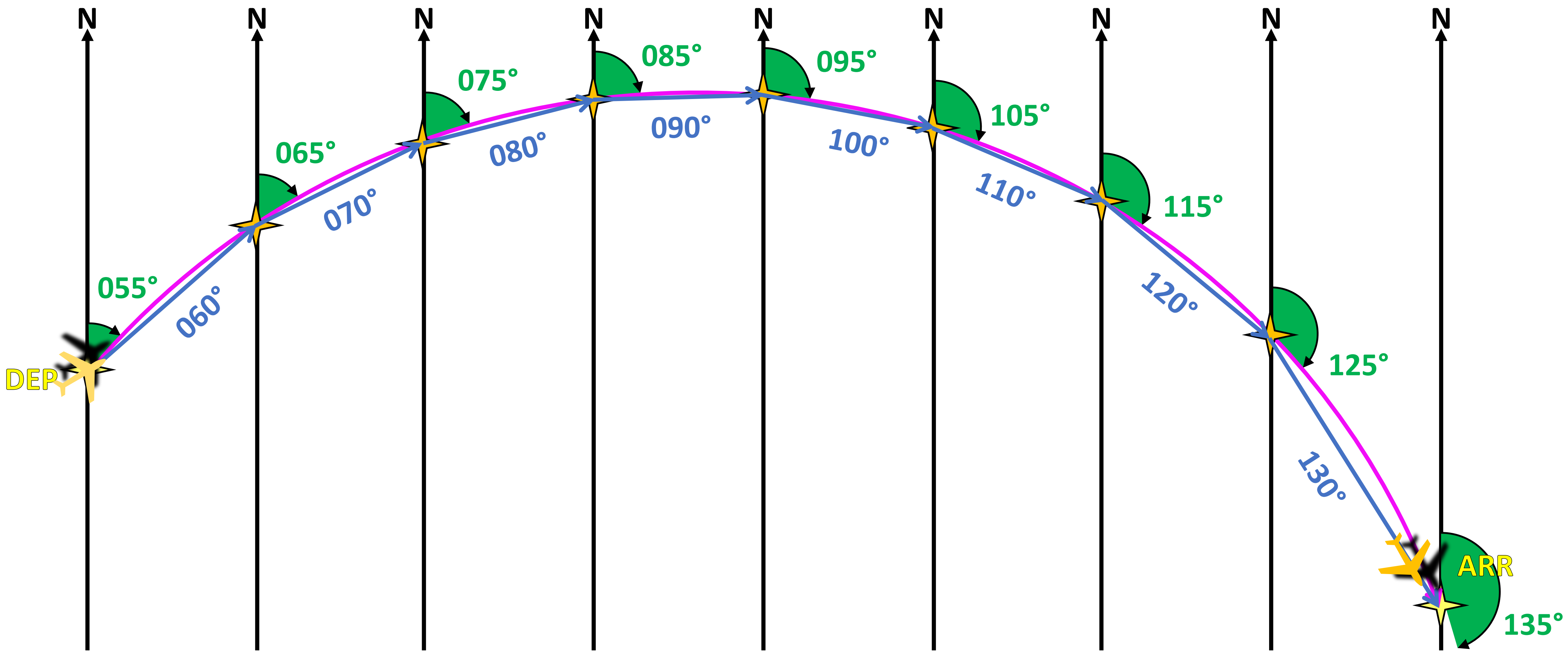
Notice that the **Rhumb Line Track** between two points is always equal to the **Average Great Circle Track** or the **Great Circle Track halfway** between these two points.

To remain even closer to the Great Circle from Departure to Arrival, we can more 'waypoints' ✦ on the Great Circle between the Departure and the Arrival point, and follow the Rhumb Lines between each point.



To determine the angle of the Rhumb Line Track to follow between two points, we need to measure the angle halfway or in the middle between these two points.

The easiest way to navigate between two points, is to follow the **Rhumb Line Track** between these two points. The more point we add waypoints ✦ on the route, the closer we approximate the **Great Circle** between these points and the shorter the distance.

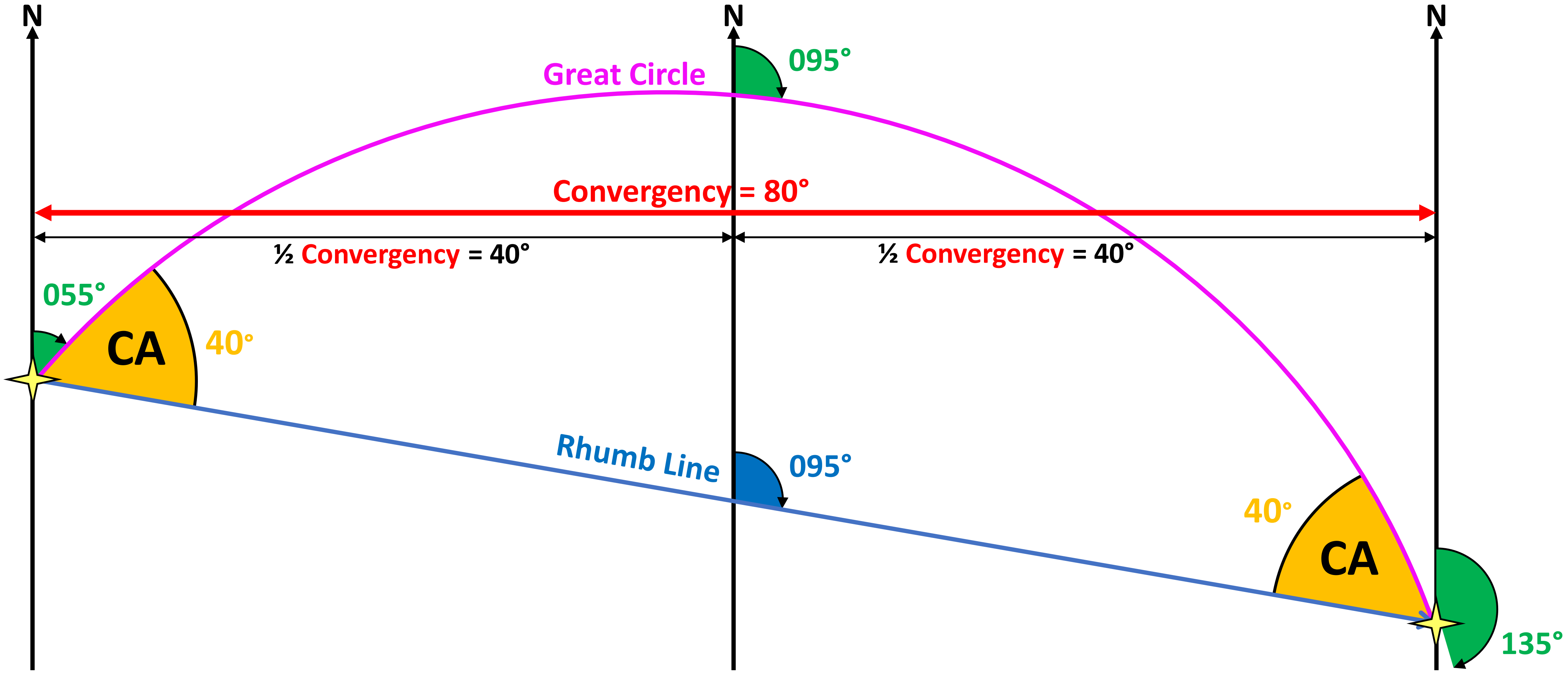


However, adding more waypoints ✦ demands more preparation and workload when navigating. It recommended to add a waypoints ✦ where the **Great Circle Track** doesn't change more than 10° (convergency no more than 10°).

We saw that the **Rhumb Line Track** between two points is equal to **Great Circle Track halfway** between these points.

Between two points, the **Great Circle Track** changes by the convergency between the meridians of these points.

Although, halfway between these two points, the **Great Circle Track halfway** changes by **half** of the **convergency** to become the value of the **Rhumb Line Track**.



So the difference between the **Great Circle** and the **Rumble Line** between two points, is half of the **convergency** between these two points. This angle is called the **Conversion Angle (CA)**.

→ **Conversion Angle (CA)** = ½ **Convergency** = $\frac{\text{Convergency}}{2}$

Since the **Rhumb Line Track** between two points is equal to **Great Circle Track halfway** between these points.
This means that if we know the **Rhumb Line Track** between two points, we also know the **Great Circle Track halfway** between these points.
So we can determine the Great Circle Track at any place between these two points because we already know the Great Circle Track at a given place (halfway)

Eg. *The Rhumb Line from A (50°N 10°E) to B (56°N 30°E) is 060, what is the initial Great Circle Track at A?*

We know that the route between two point will change by the convergency of the meridians at these two points.

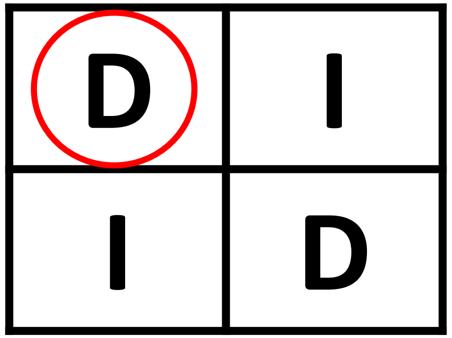
Convergency [A/B] = ChLONG [A/B] x sin Mean LAT
Convergency [A/B] = 20° x sin 53°
Convergency [A/B] ≈ 16°

We know that the Rhumb Line between A and B is 060°, **this means that the Great Circle Track halfway between A and B is 060°.**

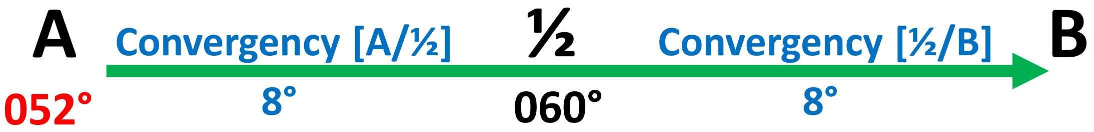
So now that we have a Great Circle Track in one location, we can calculate any Great Circle Track at any location between A and B.

→ From A to halfway, the Great Circle Track will change by ½ of the convergency, so **16°/2 = 8°**

So now let's see how the Great Circle track increases and decreases.



Since the location of the point A is West of the point B, so A is also West of the halfway point, and we are in the Northern Hemisphere, so the GCT at halfway point increases compared to the GCT at A, and it will increase by the ½ of the convergency between A and B.



Therefore, the GCT at A is 052°

Facts to know…

Fact 1

Remember that a parallel of latitude is a circle around the pole that cuts all the meridians at 90°.

When two points are on the same latitude, it means that the Rhumb Line between these two points is their own parallel of latitude.

So the Rhumb Line Track between two points on the same latitude is **090°** or **270°**.

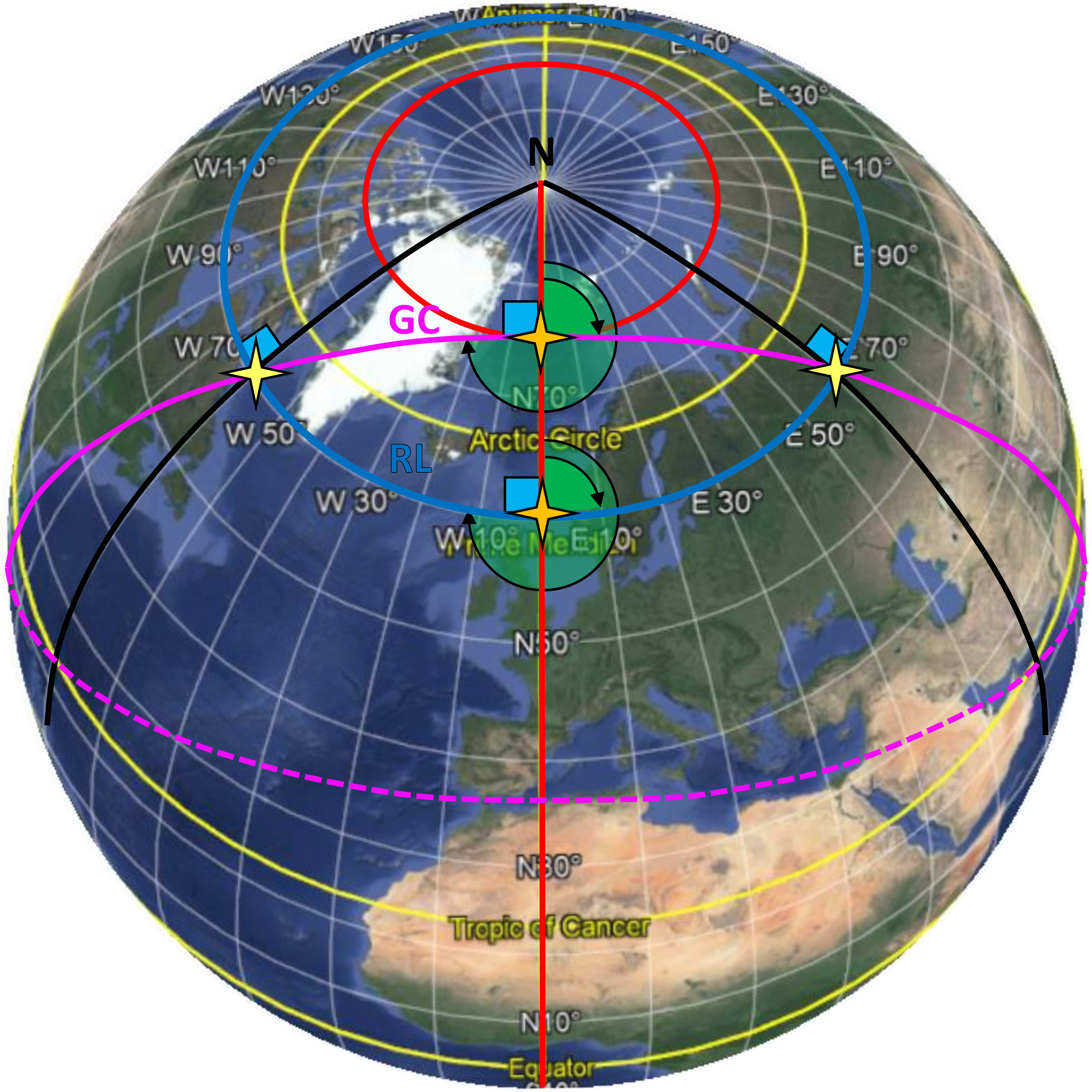
Which means that the Great Circle Track halfway between these two points is also **090°** or **270°**.

So the Great Circle Track halfway between two points on the same latitude is **090°** or **270°**.

To remember!

When two points are on the same latitude,

- the Rhumb Line Track is **090°** or **270°**
- the Great Circle Track halfway is **090°** or **270°**



Fact 2

Remember that **when passing the highest latitude, the Great Circle Track is 090° or 270°**. So when **two points are on the same latitude**, since the Great Circle Track halfway is **090°** or **270°**, the highest latitude is reached halfway between these points.

***Eg**. A route is flown from A (60°N 10°E) to B (60°N 20°W), what is the final Great Circle Track at B?*

We know that the route between two point will change by the convergency of the meridians at these two points.

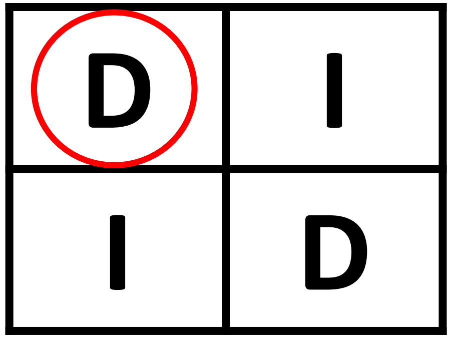
Convergency [A/B] = ChLONG [A/B] x sin Mean LAT
Convergency [A/B] = 30° x sin 60°
Convergency [A/B] ≈ 26°

- A and B are on the same latitude, it means that the Rhumb Line between A and B is their own parallel of latitude that cuts all meridians at 90°. So the the Rhumb Line Track between A and B is 090° or 270°
- The route between A and B is westerly, so the Rhumb Line Track between A and B is 270°
- If the Rhumb Line Track between A and B is 270°, **so the Great Circle Track halfway between A and B is 270°**

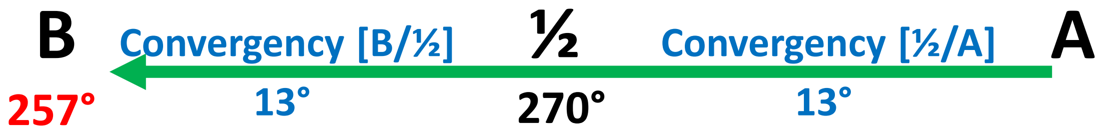
So now that we have a Great Circle Track in one location, we can calculate any Great Circle Track at any location between A and B.

→ From halfway to B, the Great Circle Track will change by ½ of the convergency, so **26°/2 = 13°**

So now let's see how the Great Circle track increases and decreases.



Since the location of the point B is West of the point A, so B is also West of the halfway point, and we are in the Northern Hemisphere, so the GCT at B point decreases compared to the GCT halfway, and it will decrease by the ½ of the convergency between A and B.



Therefore, the GCT at B is 257°

Exercises:

a. The The Rhumb Line from A (45°S 50°E) to B (53°S 63°E) is 070, what is the initial Great Circle Track at A?

b. If the initial true great circle track from C (N55° E/W000°) to D (N54° E010°) is 100°, what is the Rhumb Line track at C?

c. (1) What is the rhumb line track from E (58°12'N 004°00'W) to F (58°12'N 008°00'E)? (2) What is the initial great circle track from F to E?

d. The following waypoints are entered into an inertial navigation system (INS):
WPT 1: (53°08'N030W) WPT 2: (53°08'N020W) WPT 3: (53°08'N010W). The inertial navigation is connected to the automatic pilot on the route WPT1 – WPT2 – WPT3. By how much will approximately the track change on passing WPT2?

The correction is in the next page…

Exercise *(corrected)*:

a. The Rhumb Line from A (45°S 50°E) to B (53°S 63°E) is 070, what is the initial Great Circle Track at A?

We know that the route between two points will change by the convergency of the meridians at these two points.

Convergency [A/B] = ChLONG [A/B] x sin Mean LAT

Convergency [A/B] = 13° x sin 49°

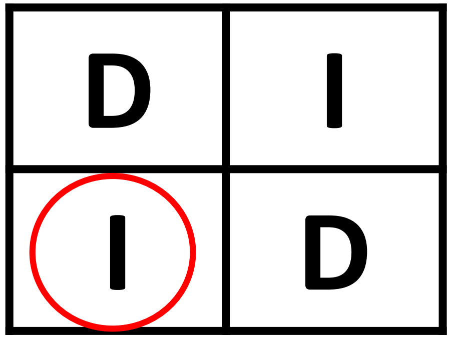
Convergency [A/B] ≈ 10°

We know that the Rhumb Line between A and B is 070°, **this means that the Great Circle Track halfway between A and B is 070°.**

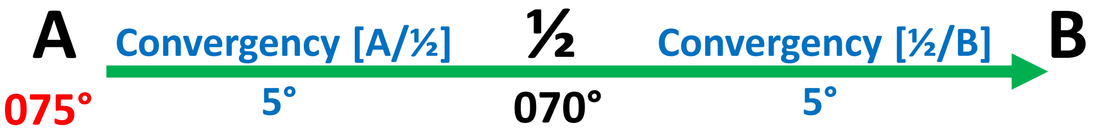
So now that we have a Great Circle Track in one location, we can calculate any Great Circle Track at any location between A and B.

→ From A to halfway, the Great Circle Track will change by ½ of the convergency, so **10°/2 = 5°**

So now let's see how the Great Circle track increases and decreases.



Since the location of the point A is West of the point B, so A is also West of the halfway point, and we are in the Southern Hemisphere, so the GCT at halfway point decreases compared to the GCT at A, and it will decrease by the ½ of the convergency between A and B.



Therefore, the GCT at A is 075°

Exercise *(corrected)*:

b. If the initial true great circle track from C (N55° E/W000°) to D (N54° E010°) is 100°, what is the Rhumb Line track at C?

We know that the route between two points will change by the convergency of the meridians at these two points.

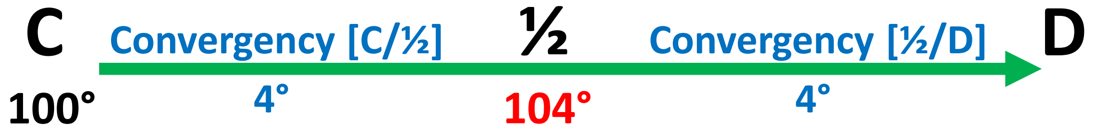
Convergency [C/D] = ChLONG [C/D] x sin Mean LAT
Convergency [C/D] = 10° x sin 54.5°
Convergency [C/D] ≈ 8°

- The Rhumb Line Track between two points is the same angle everywhere
- The Rhumb Line between C and D is equal to the Great Circle Track halfway between C and D
- The Great Circle Track between C and D halfway will change by ½ of the convergency between C and D, so by **4° (8°/2 = 4°)**

So now let's see how the Great Circle track increases and decreases.

D	I
I	D

Since the location of the point C is West of the point D, so the halfway point is East of C, and we are in the Northern Hemisphere, so the GCT at halfway point increases compared to the GCT at C, and it will increase by the ½ of the convergency between C and D.



Therefore, the RLT is 104°

Exercise *(corrected)*:

c.

1. What is the rhumb line track from E (58°12’N 004°00’W) to F (58°12’N 008°00’E)?
2. What is the initial great circle track from F to E?

Answer 1.

- A and B are on the same latitude, it means that the Rhumb Line between E and F is their own parallel of latitude that cuts all at 90°. So the the Rhumb Line Track between A and B is 090° or 270°
- The route between E and F is easterly, **so the Rhumb Line Track between E and F is 090°**

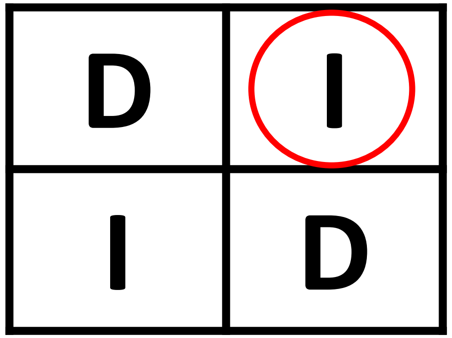
Answer 2.

We know that the route between two points will change by the convergency of the meridians at these two points.

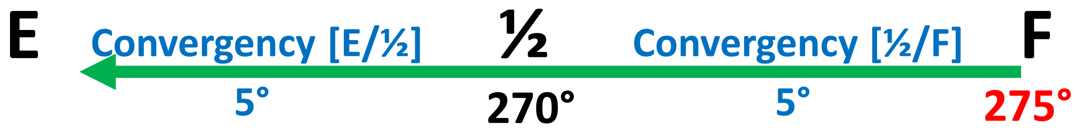
Convergency [E/F] = ChLONG [E/F] x sin Mean LAT = 12° x sin 58.2°≈ **10°**

→ From F to halfway, the Great Circle Track will change by ½ of the convergency, so **10°/2 = 5°**

If the Rhumb Line Track between E and F is 090°, so the Rhumb Line between F and E is 270°, **this means that the Great Circle Track halfway between F and E is 270°.**



Since the location of the point is East of the point E, so F is also East of the halfway point, and we are in the Northern Hemisphere, so the GCT at F decreases compared to the GCT at halfway point, and it will decrease by the ½ of the convergency between E and F.



Therefore, the inital GCT at E is 275°

Advanced Exercise *(corrected)*:

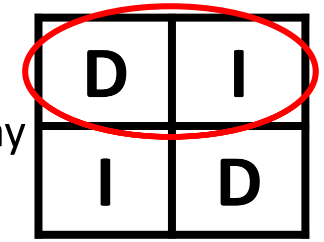
d. The following waypoints are entered into an inertial navigation system (INS):
WPT 1: (53°08’N030W) WPT 2: (53°08’N020W) WPT 3: (53°08’N010W). The inertial navigation is connected to the automatic pilot on the route WPT1 – WPT2 – WPT3. By how much will approximately the track change on passing WPT2?

It is important to understand that the INS allows the aircraft to maintain the Great Circle between 2 points, since 3 points have been inserted in the INS, the aircraft will flyover these 3 points. So it will fly 2 Great Circles, Great Circle 1 from WPT1 to WPT2, then at WPT2 it will turn to engage on the next Great Circle 2 between WPT2 and WPT3.
The three points are on the same latitude and separated by 10° change of longitudes. The tracks angle of the 2 Great Circle will be the same.

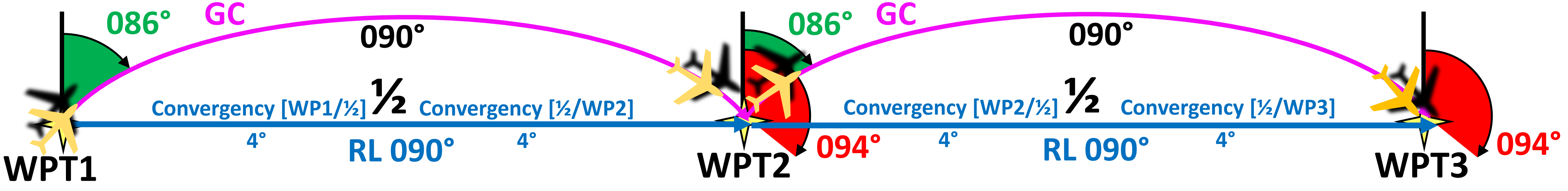
We know that the route between two points will change by the convergency of the meridians at these two points.

Convergency [WPT1/WPT2] = Convergency [WPT2/WPT3] = ChLONG x sin Mean LAT = 10° x sin 53°≈ **8°**

The 3 points are on the same latitude, and the direction is Easterly, it means that the Rhumb Line [WPT1/WPT2] and [WPT2/WPT3] is 090°, **so the Great Circle halfway [WPT1/WPT2] and [WPT2/WPT3] is 090°**.



For the Great Circle [WPT1/WPT2] and the Great Circle [WPT2/WPT3]
→ The **initial Great Circle Track** decreases by ½ of the convergency (**4°**) compared to the Great Circle Track halfway
→ The **final Great Circle Track** increases by ½ of the convergency (**4°**) compared to the Great Circle Track halfway



Therefore, when passing the WPT2, the track decreases by 8°